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ADAPTIVE ROBUST ESTIMATION OF LOCATION AND SCALE PARAMETERS OF SYMMETRIC POPULATIONS

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In 1972 Harter proposed, as adaptive robust estimators of the mean (μ) and standard deviation (σ) of a symmetric population, the maximum likelihood (ML) estimators for the uniform (U), normal (N) or double exponential (D) population, according as $K < 2.2$, $2.2 \leq K \leq 3.8$, where K is the sample kurtosis. Various authors have studied the use of other criteria, based on Hogg's Q statistic and the likelihood function, and the effect of varying the critical values. Critical values of several criteria, for equal Type I and Type II error rates

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Uniform (U), Normal (N), and Double (D)

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(probabilities of misclassification), have now been established for sample sizes n = 8 (4) 24 by means of a Monte Carlo study based on 5000 random samples of each size from each of the above populations (U, N and D). Mean square errors of the adaptive estimates are compared with those of the ML estimates if the population from which each sample came is known, and the effect of debiasing the ML estimates of σ is studied. Adaptive estimation of the canonical scale parameter Fσ, where the factor F is defined as the multiplier of σ such that Fσ is the 97.5% point of a population symmetric about zero, is also considered. Monte Carlo studies have also been conducted to determine the performance of the various criteria when applied to an independent set of random samples (obtained by using a different seed for the random number generator) from U, N and D and to random samples from several other symmetric populations, for the above values of n and for the intermediate values n = 10 (4) 22, with critical values of the criteria determined by interpolation.

sigma

F sigma

the standard deviation (sigma)

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PREFACE

During the past several years, the first two authors have performed extensive research on adaptive robust estimation and have served as sponsor and faculty advisor, respectively, for several Air Force Institute of Technology M.S. theses on the subject. The work documented here represents an extension of all those efforts, especially that of the third author in the latest of the series of AFIT theses, which reported the results of a small Monte Carlo study of the performances of various estimators. The authors wish to thank Lt Michael Himmelberg of the ASD Computer Center for performing, on the CDC 6600 computer, the much more extensive Monte Carlo study which is described in Section III and whose results are reported in Appendix A of this report.

The work of the first author was performed under work unit 2304N101, Order Statistics and their Use in Testing and Estimation. This is the final report on that in-house work unit.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF TABLES (CONTINUED)

LIST OF TABLES (CONTINUED)

SECTION I

INTRODUCTION

One of the fundamental problems of statistics is that of estimating one or more population parameters on the basis of information contained in a random sample. Foremost among the parameters to be estimated are the location and scale parameters.

During the greater part of the nineteenth century, most statisticians, following Gauss (1809), pursued the dogma of normality, believing (or at least behaving as if they believed) that all errors in observations are normally distributed. The location parameter (mean) μ and the scale parameter (standard deviation) σ are sufficient to specify a normal distribution completely. If the distribution is indeed normal, the sample mean and standard deviation are maximum likelihood estimators of the corresponding population parameters.

In the 1880's, Edgeworth and Newcomb became concerned about the consequences of using the sample mean and standard deviation and the method of least squares, all based on the normal distribution, when the assumption of normality is not valid. Edgeworth (1886) declared that, on the grounds of precision, the arithmetic mean is superior for the normal distribution and others near it, but the median is better for long-tailed distributions, i.e. "when the apex of the curve is very high and its extremities very much extended." Newcomb (1886), after examining a collection of 684 residual errors based on observations of a transit of Mercury, developed an estimator based on a mixture of normal density functions. Unfortunately, even though many statisticians

recognized the problem, there was little progress in solving it during the following 60 years. In the late 1940's, however, Tukey and his colleagues in the Statistical Research Group at Princeton began to offer practical solutions to the problem by establishing several properties of alternative estimators.

Box (1953) coined the term "robustness." A robust procedure is one that still performs very well under moderate changes in assumptions concerning the underlying distribution. Box found, in particular, that analysis-of-variance tests are quite robust to departures from the assumption of homogeneity of variance. The concept of robustness soon came to be applied to estimation procedures as well as to test procedures. Starting in the early 1960's, several authors, including Hodges & Lehmann (1963) and Huber (1964), proposed various robust estimators of location parameters of symmetric distributions. Some of them also considered the problem of robust estimation of scale parameters, which involves certain complications not present in the case of location parameters. In the first place, as Huber pointed out, the standard deviation σ does not have the same properties for all distributions, since the interval $(\mu - k\sigma, \mu + k\sigma)$, where k is a constant, includes widely varying proportions of various populations, so that there is no natural "canonical" scale parameter to be estimated. Secondly, the sample mean and other common estimators of the location parameter of a symmetric distribution (median, mode, and midrange) are unbiased, but the sample standard deviation and other common estimators of the scale parameter are biased; moreover, the magnitude of the bias depends on the underlying distribution.

Adaptive robust estimators are estimators which are designed to achieve some degree of robustness by varying the estimation procedure according to the value of some measure of the sample indicative of the population type. Hogg (1967) proposed an adaptive robust estimator of the location parameter of a symmetric population based on varying the estimation procedure according to the value of the sample kurtosis K. Harter (1972) proposed another such estimator based on classifying the sample as having come from a uniform, normal, or double exponential population according as $K < K_L$, $K_L \leq K \leq K_U$, or $K > K_U$, with K_L and K_U tentatively taken to be 2.2 and 3.8, respectively, and then using the appropriate maximum likelihood estimators of μ and σ. At about the same time, Hogg (1972) proposed an alternative criterion based on the value of the statistic $Q = [\bar{U}(\alpha) - \bar{L}(\alpha)]/[\bar{U}(\beta) - \bar{L}(\beta)]$ where $\bar{U}(\beta)[\bar{L}(\beta)]$ is the average of the largest [smallest] $n\beta$ order statistics (where n is the sample size), and Hogg, Uthoff, Randles & Davenport (1972) proposed still another criterion based on maximizing weighted likelihood functions.

In what is commonly known as the "Princeton study," Andrews, Bickel, Hampel, Huber, Rogers & Tukey (1972) made a comprehensive theoretical and Monte Carlo study of robust estimates of location. On a smaller scale, a similar study of robust estimates of both location and scale parameters has been carried out in a series of Air Force Institute of Technology Master's theses by Caso (1972), Jorgenson (1973), Forth (1974), Rugg (1974), Almquist (1975), and Curry (1977). In this series, both nonadaptive and adaptive procedures have been considered, with emphasis on the latter. The effect of varying the critical values for the K, Q and likelihood criteria

has been considered, along with objective methods of choosing the critical values. The purpose of the present work is to summarize and extend the results, to study the performance of adaptive robust estimators of location and scale parameters for samples from a broad spectrum of symmetric distributions, and to make recommendations concerning their use.

Section II will deal with the adaptive robust estimators and the measures of their performance to be considered. Section III will describe the various phases of the Monte Carlo study conducted to establish the critical values of the criteria and compare the performance of the estimators. Section IV will present conclusions and recommendations.

SECTION II

ESTIMATORS AND MEASURES OF PERFORMANCE

All of the adaptive robust estimation procedures to be considered will involve classifying the sample as having come from a uniform, normal, or double exponential population and then using the maximum likelihood estimators of the location and scale parameters for the appropriate population. If the sample actually came from the population into which it is classified (or from any symmetric population), the maximum likelihood estimator $\hat{\mu}$ of the location parameter μ (the population mean) is unbiased, but the maximum likelihood estimator $\hat{\sigma}$ of the scale parameter σ (the population standard deviation) is biased. The debiased maximum likelihood estimator of the scale parameter, $\bar{\sigma} = C\hat{\sigma}$, will be considered in addition to $\hat{\sigma}$. The debiasing factors C_U, C_N and C_D for

samples of size n = 8(2)24 from uniform, normal and double exponential populations, respectively, are:

Sample Size, n	$C_U = (n+1)/(n-1)$	$C_N = \sqrt{n/(n-1)}/c_2$	$C_D = n/E(V_n)$
8	9/7 = 1.286	$\sqrt{8/7}/.9650 = 1.108$	8/7.449 = 1.074
10	11/9 = 1.222	$\sqrt{10/9}/.9727 = 1.084$	10/9.449 = 1.058
12	13/11 = 1.182	$\sqrt{12/11}/.9776 = 1.068$	12/11.45 = 1.048
14	15/13 = 1.154	$\sqrt{14/13}/.9810 = 1.058$	14/13.45 = 1.041
16	17/15 = 1.133	$\sqrt{16/15}/.9835 = 1.050$	16/15.45 = 1.036
18	19/17 = 1.118	$\sqrt{18/17}/.9854 = 1.044$	18/17.45 = 1.032
20	21/19 = 1.105	$\sqrt{20/19}/.9869 = 1.040$	20/19.46 = 1.028
22	23/21 = 1.095	$\sqrt{22/21}/.9882 = 1.036$	22/21.51 = 1.025
24	25/23 = 1.087	$\sqrt{24/23}/.9892 = 1.033$	24/23.51 = 1.021

Values of c_2 used in obtaining the above table were taken from a table by Harter (1970) and those of $E(V_n)$ from a table (for $n \leq 20$) and an asymptotic approximation (for n > 20) by Bain & Engelhardt (1973).

As mentioned in the introduction, another difficulty with estimating the scale parameter is that the standard deviation σ, which will be called the scale parameter, does not have the same properties for different populations. In particular, the percentage of the population contained in the interval $(\mu - k\sigma, \mu + k\sigma)$, where k is a constant, may vary widely from one population to another. For example, for k = 1, the interval $(\mu - \sigma, \mu + \sigma)$ contains 57.74% of the uniform population, 68.27% of the normal population, and 75.69% of the double exponential population. These proportions are approximately equalized for k = 1.5,

and their order is reversed for $k = 2$. The interval $(\mu - 1.5\sigma, \mu + 1.5\sigma)$ contains 86.60% of the uniform population, 86.64% of the normal population, and 88.01% of the double exponential population. The interval $(\mu - 2\sigma, \mu + 2\sigma)$ contains 100% of the uniform population, 95.45% of the normal population, and 94.09% of the double exponential population. In an effort to resolve this problem, estimates $F\hat{\sigma}$ and $F\bar{\sigma}$ of the canonical scale parameter $F\sigma$ will be considered in addition to $\hat{\sigma}$ and $\bar{\sigma}$. The canonical scale factor F, which is defined as the value such that $(\mu - F\sigma, \mu + F\sigma)$ contains 95% of the population, so that $\mu + F\sigma$ is the 97.5% point of a symmetric population, is 1.64545, 1.95996 and 2.11833 for uniform, normal and double exponential populations, respectively.

The maximum likelihood estimators of the location parameter (population mean) μ are the sample midrange for the uniform population, the sample mean for the normal population, and the sample median for the double exponential population. These estimators are given by the following equations:

$$\hat{\mu}_U = (x_1 + x_n)/2 \tag{1}$$

$$\hat{\mu}_N = \textstyle\sum_{i=1}^{n} x_i/n \tag{2}$$

$$\hat{\mu}_D = \begin{cases} x_{(n+1)/2} & (n \text{ odd}) \\ (x_{n/2} + x_{n/2+1})/2 & (n \text{ even}) \end{cases} \tag{3}$$

where $x_1 \leq x_2 \leq \ldots \leq x_n$ are the <u>ordered</u> sample values.

The maximum likelihood estimators of the scale parameter (population standard deviation) σ are the sample range divided by $2\sqrt{3}$ (or the sample semirange$/\sqrt{3}$) for the uniform population, the sample standard deviation

for the normal population, and $\sqrt{2}$ times the mean deviation from the sample median for the double exponential population. These estimators are given by the following equations:

$$\hat{\sigma}_U = (x_n - x_1)/2\sqrt{3} \tag{4}$$

$$\hat{\sigma}_N = \sqrt{\textstyle\sum_{i=1}^{n} (x_i - \hat{\mu}_N)^2/n} \tag{5}$$

$$\hat{\sigma}_D = \sqrt{2} \textstyle\sum_{i=1}^{n} |x_i - \hat{\mu}_D|/n \tag{6}$$

The adaptive robust estimators are $\hat{\mu}_U$ and $\hat{\sigma}_U$ if the sample is classified as having come from a uniform population, $\hat{\mu}_N$ and $\hat{\sigma}_N$ if it is classified as having come from a normal population, and $\hat{\mu}_D$ and $\hat{\sigma}_D$ if it is classified as having come from a double exponential population.

The criteria used in classifying a sample as having come from a uniform, normal or double exponential population are based on the sample kurtosis K, the sample value of Hogg's Q statistic, or the sample values of the *likelihood* for the three populations. These criteria are as follows:

(a) Criterion based on the sample kurtosis

$$K = \textstyle\sum_{i=1}^{n} (x_i - \hat{\mu}_N)^4/n\hat{\sigma}_N^4 \tag{7}$$

If $K < K_L$, classify the sample as U (uniform)

If $K_L \leq K \leq K_U$, classify the sample as N (normal)

If $K > K_U$, classify the sample as D (double exponential)

where the critical values K_L and K_U <u>for each sample size</u> may be calculated in either of two ways:

(1) Choose $K_L = K_{L1}$ so that the proportion of N's (samples actually coming from a normal population) for which $K < K_{L1}$ equals the proportion of U's (samples actually coming from a uniform population) for which $K \geq K_{L1}$. Choose $K_U = K_{U1}$ so that the proportion of N's for which $K > K_{U1}$ equals the proportion of D's (samples actually coming from a double exponential population) for which $K \leq K_{U1}$.

(2) Choose $K_L = K_{L2}$ so that the proportion of N's and D's for which $K < K_{L2}$ equals the proportion of U's for which $K \geq K_{L2}$. Choose $K_U = K_{U2}$ so that the proportion of U's and N's for which $K > K_{U2}$ equals the proportion of D's for which $K \leq K_{U2}$.

(b) Criterion based on Hogg's statistic

$$Q = [\overline{U}(.04) - \overline{L}(.04)]/[\overline{U}(.5) - \overline{L}(.5)] \tag{8}$$

(for convenience we have taken α = .04 instead of .05 as suggested by Hogg).

If $Q < Q_L$, classify the sample as U

If $Q_L \leq Q \leq Q_U$, classify the sample as N

If $Q > Q_U$, classify the sample as D

where the critical values Q_L and Q_U for each sample size may be calculated in either of two ways:

(1) by replacing K by Q in (a)(1) above

(2) by replacing K by Q in (a)(2) above

(c) Criterion based on the largest likelihood:

If $L_U > L_N$ and $L_U > L_D$, classify the sample as U

If $L_N \geq L_U$ and $L_N \geq L_D$, classify the sample as N

If $L_D \geq L_U$ and $L_D > L_N$, classify the sample as D

where the likelihood functions are given by the equations

$$L_U = (1/2\ \hat{\sigma}_U \sqrt{3})^n \tag{9}$$

$$L_N = (1/\hat{\sigma}_N \sqrt{2\pi})^n \exp\left[- \sum_{i=1}^{n}(x_i - \hat{\mu}_N)^2/2\hat{\sigma}_n^2\right] \tag{10}$$

$$L_D = (1/\hat{\sigma}_D\sqrt{2})^n \exp\left[- \sqrt{2} \sum_{i=1}^{n}|x_i - \hat{\mu}_D|/\hat{\sigma}_D\right] \tag{11}$$

(d) Criterion based on the ratio of the two larger likelihoods:

If $L_U > L_D$, $L_N \geq L_D$ and $\lambda_1 < \lambda_1^*$, classify the sample as U

If $L_U > L_D$, $L_N \geq L_D$ and $\lambda_1 \geq \lambda_1^*$, classify the sample as N

If $L_U > L_N$, $L_D > L_N$ and $\lambda_2 < \lambda_2^*$, classify the sample as U

If $L_U > L_N$, $L_D > L_N$ and $\lambda_2 \geq \lambda_2^*$, classify the sample as D

If $L_N \geq L_U$, $L_D \geq L_U$ and $\lambda_3 \leq \lambda_3^*$, classify the sample as N

If $L_N \geq L_U$, $L_D \geq L_U$ and $\lambda_3 > \lambda_3^*$, classify the sample as D

where

$$\lambda_1 = (L_N/L_U)^{1/n},\ \lambda_2 = (L_D/L_U)^{1/n},\ \lambda_3 = (L_D/L_N)^{1/n} \tag{12}$$

and where the critical values λ_1^*, λ_2^* and λ_3^* <u>for each sample size</u> may be calculated in any of three ways:

(1) Choose $\lambda_1^* = \lambda_{11}^*$ so that, when $L_U > L_D$ and $L_N \geq L_D$, the proportion of N's for which $\lambda_1 < \lambda_{11}^*$ equals the proportion of U's for which $\lambda_1 \geq \lambda_{11}^*$. Choose $\lambda_2^* = \lambda_{21}^*$ so that, when $L_U > L_N$ and $L_D > L_N$, the proportion of D's for which

$\lambda_2 < \lambda_{21}^*$ equals the proportion of U's for which $\lambda_2 \geq \lambda_{21}^*$. Choose $\lambda_3^* = \lambda_{31}^*$ so that, when $L_N \geq L_U$ and $L_D \geq L_U$, the proportion of D's for which $\lambda_3 \leq \lambda_{31}^*$ equals the proportion of N's for which $\lambda_3 > \lambda_{31}^*$.

(2) Choose $\lambda_1^* = \lambda_{12}^*$ so that, when $L_U > L_D$ and $L_N \geq L_D$, the proportion of N's and D's for which $\lambda_1 < \lambda_{12}^*$ equals the proportion of U's for which $\lambda_1 \geq \lambda_{12}^*$. Choose $\lambda_2^* = \lambda_{22}^*$ so that, when $L_U > L_N$ and $L_D > L_N$, the proportion of D's and N's for which $\lambda_2 < \lambda_{22}^*$ equals the proportion of U's and N's for which $\lambda_2 \geq \lambda_{22}^*$. Choose $\lambda_3^* = \lambda_{32}^*$ so that, when $L_N \geq L_U$ and $L_D \geq L_U$, the proportion of D's for which $\lambda_3 \leq \lambda_{32}^*$ equals the proportion of U's and N's for which $\lambda_3 > \lambda_{32}^*$.

(3) Choose $\lambda_1^* = \lambda_{13}^*$ so that the proportion of N's for which $\lambda_1 < \lambda_{13}^*$ equals the proportion of U's for which $\lambda_1 \geq \lambda_{13}^*$. Choose $\lambda_2^* = \lambda_{23}^*$ so that the proportion of D's for which $\lambda_2 < \lambda_{23}^*$ equals the proportion of U's for which $\lambda_2 \geq \lambda_{23}^*$. Choose $\lambda_3^* = \lambda_{33}^*$ so that the proportion of D's for which $\lambda_3 \leq \lambda_{33}^*$ equals the proportion of N's for which $\lambda_3 > \lambda_{33}^*$.

(e) Criterion based on the dominant likelihood:

If $\lambda_1 < \lambda_{13}^*$ and $\lambda_2 < \lambda_{23}^*$, classify the sample as U

If $\lambda_1 \geq \lambda_{13}^*$ and $\lambda_3 \leq \lambda_{33}^*$, classify the sample as N

If $\lambda_2 \geq \lambda_{23}^*$ and $\lambda_3 > \lambda_{33}^*$, classify the sample as D

If none of the above pairs of inequalities holds, classify the sample as N.

The performance of an adaptive robust estimator may be measured in various ways, of which only two will be considered here. Of two or more adaptive estimators under consideration, we may prefer the one which (a) maximizes the number (or proportion) of samples classified correctly or (b) minimizes the mean square error (or maximizes the efficiency relative to the maximum likelihood estimator if the population is known). For adaptive robust estimators of the type under consideration, (a) is applicable only to samples from uniform, normal and/or double exponential populations, since otherwise correct classification is impossible, while (b) may be applied to samples from any population.

SECTION III

MONTE CARLO STUDY

A Monte Carlo simulation of the estimators based on the criteria [(a)(1), (a)(2), (b)(1), (b)(2), (c), (d)(1), (d)(2), (d)(3) and (e)] outlined in Section II was conducted in four phases. In Phase I, 5000 samples of each size n = 8(4)24 were drawn from each of the following populations (standardized so as to have mean zero and standard deviation one): uniform, normal and double exponential. The probability density functions of these populations are as follows:

(a) Uniform: $f_U(x) = 1/2\sqrt{3}, \quad (-\sqrt{3}, \sqrt{3})$ (13)

(b) Normal: $f_N(x) = \exp(-x^2/2)/\sqrt{2\pi}, \quad (-\infty, \infty)$ (14)

(c) Double Exponential: $f_D(x) = \exp(-\sqrt{2}|x|)/\sqrt{2}, \quad (-\infty, \infty)$ (15)

The standardized random variables were obtained by generating random variables uniformly distributed between 0 and 1 by use of the library subroutine RANF on the CDC 6600 computer and transforming them as follows:

(a) Uniform: If r is uniform between 0 and 1, then

$$U = \sqrt{3}\ (2r - 1) \tag{16}$$

is standardized uniform.

(b) Normal: If r_1 and r_2 are independent uniform random variables between 0 and 1, then [see Box & Muller (1958)]

$$N_1 = \sqrt{-2\ell n(r_2)}\ \cos(2\pi r_1) \tag{17}$$

and

$$N_2 = \sqrt{-2\ell n(r_2)}\ \sin(2\pi r_1) \tag{18}$$

are standardized normal.

(c) Double Exponential: If r is uniform between 0 and 1, then

$$D = \begin{cases} \ell n(2r)/\sqrt{2}, & 0 < r < .5 \\ -\ell n(2-2r)/\sqrt{2}, & .5 \leq r < 1 \end{cases} \tag{19}$$

is standardized double exponential.

The values of $\hat{\mu}_U$, $\hat{\mu}_N$, $\hat{\mu}_D$, $\hat{\sigma}_U$, $\hat{\sigma}_N$, $\hat{\sigma}_D$, K, Q, L_U, L_N, L_D, λ_1, λ_2 and λ_3 were then computed for each sample by use of Equations (1)-(12). For each sample size, the 15000 sample values of K (5000 from each population) were used to determine the critical values K_{L1} and K_{U1}

satisfying the criterion (a)(1) and the critical values K_{L2} and K_{U2} satisfying the criterion (a)(2), as defined in Section II. Similarly, the sample values of Q were used to determine the critical values Q_{L1} and Q_{U1} satisfying the criterion (b)(1) and the critical values Q_{L2} and Q_{U2} satisfying the criterion (b)(2), and the sample values of λ_1, λ_2 and λ_3 were used to determine the critical values λ^*_{11}, λ^*_{21} and λ^*_{31} satisfying the criterion (d)(1), the critical values λ^*_{21}, λ^*_{22} and λ^*_{32} satisfying the criterion (d)(2), and the critical values λ^*_{31}, λ^*_{32} and λ^*_{33} satisfying the criteria (d)(3) and (e). These critical values for each combination of criterion and sample size are shown in Table 1 of Appendix A. As noted in the table, no critical values are required for criterion (c), which classifies a sample as having come from the population (U, N or D) for which the sample likelihood is greatest. These criteria were used, along with the critical values shown in Table 1, to classify each sample as U, N, D (having come from a uniform, normal or double exponential population, respectively). Tables 2-6 of Appendix A are contingency tables showing the number of samples from each population (U, N and D) classified by each criterion as U, N and D, for n = 8(4)24, respectively.

The adaptive robust estimates $\hat{\mu}_C$, $\hat{\sigma}_C$, $\bar{\sigma}_C = C_C\hat{\sigma}_C$, $F_C\hat{\sigma}_C$ and $F_C\bar{\sigma}_C$ were computed for each sample, where the subscript C is U, N or D according as the sample was classified as U, N or D, where C_U, C_N and C_D are tabulated in the first paragraph of Section II, and where $F_U = 1.64545$, $F_N = 1.95996$ and $F_D = 2.11833$. The corresponding maximum likelihood (or debiased maximum likelihood) estimates $\hat{\mu}_T$, $\hat{\sigma}_T$, $\bar{\sigma}_T = C_T\hat{\sigma}_T$, $F_T\hat{\sigma}_T$ and $F_T\bar{\sigma}_T$, where the subscript T is U, N or D according as

the true population from which the sample actually came is U, N or D, were also computed. Their mean square errors are shown in Table 7 of Appendix A. The ratios of those mean square errors to the mean square errors of the corresponding adaptive robust estimates, which are the efficiencies of the adaptive robust estimates relative to the maximum likelihood (or debiased maximum likelihood) estimates when the population from which each sample actually came is known, are shown in Tables 8-12 of Appendix A for $\hat{\mu}$, $\hat{\sigma}$, $\bar{\sigma}$, $F\hat{\sigma}$ and $F\bar{\sigma}$, respectively.

Phase II of the Monte Carlo study was a repetition of Phase I, with the following differences: (1) A different "seed" was used for the random number generator in order to obtain random samples independent of those obtained in Phase I; (2) For the various criteria, the critical values which were determined in Phase I (see Table 1 of Appendix A) were used instead of determining new ones from the new samples. In Appendix A, the contingency tables giving classification vs. true population for the samples of Phase II are shown in Tables 2-6 to the right of those for the samples in Phase I. The mean square errors of the maximum likelihood (or debiased maximum likelihood) estimates (if the population is known) which were obtained in Phase II are shown in Table 7 underneath those obtained in Phase I. The efficiencies of the adaptive robust estimates relative to the corresponding maximum likelihood (or debiased maximum likelihood) estimates for the samples of Phase II are shown in Tables 8-12 to the right of those for the samples in Phase I.

Phase III of the Monte Carlo study was similar to Phase II, the difference being that samples were drawn from other symmetric populations

instead of the uniform (kurtosis $\alpha_4 = 1.8$), normal ($\alpha_4 = 3$) and double exponential ($\alpha_4 = 6$) populations. The standardized populations considered were the following: double spike ($\alpha_4 = 1$); arc sine ($\alpha_4 = 1.5$); symmetric beta with parameters p = 1.5, 2.0, 2.5, 3.0, 3.5 and 4.0 [$\alpha_4 = 2$, 15/7 = 2.143, 2.25, 7/3 = 2.333, 2.4 and 27/11 = 2.455, respectively]; Student t with degrees of freedom ν = 16, 10, 8, 7, 6, 5 [$\alpha_4 = 3.5$, 4, 4.5, 5, 6 and 9 respectively]. Parenthetically, it may be remarked that the double spike, arc sine and uniform populations are symmetric beta populations with parameters 0, 0.5 and 1.0 respectively, while the normal population is both a symmetric beta population with parameter ∞ and a Student t population with ∞ degrees of freedom. The Student t population with 6 degrees of freedom and the double exponential population both have kurtosis $\alpha_4 = 6$, but they are not the same. The standardized double spike population is a discrete population whose probability mass function is given by

$$p_{DS}(x) = (1/2)\delta_{x,-1} + (1/2)\delta_{x,1} \tag{20}$$

where δ is the Kronecker delta [$\delta_{ij} = 1$ if $i = j$ and $\delta_{ij} = 0$ if $i \neq j$]. The probability density functions of the standardized arc sine, symmetric beta and Student t populations are as follows:

(a) Arc sine: $$f_{AS}(x) = 1/\pi\sqrt{2-x^2}, \quad (-\sqrt{2}, \sqrt{\ } \tag{21}$$

(b) Symmetric beta:

$$f_{SB}(x) = \left[\Gamma(2p)/\Gamma^2(p)(2\sqrt{2p+1})^{2p-1}\right](2p+1-x^2)^{p-1}, \quad (-\sqrt{2p+1}, \sqrt{2p+1}) \tag{22}$$

(c) Student t:

$$f_{ST}(x) = \{\Gamma[(\nu+1)/2]/\Gamma(1/2)\Gamma(\nu/2)\sqrt{\nu-2}\}\left[1+x^2/(\nu-2)\right]^{-(\nu+1)/2}, \quad (-\infty, \infty) \tag{23}$$

The standardized random variables for the double spike and arc sine populations were obtained by generating random variables uniformly distributed between 0 and 1 by use of the library subroutine RANF on the CDC 6600 computer and transforming them as follows:

(a) Double spike: If r is uniform between 0 and 1, then

$$DS = \begin{cases} -1, & r \leq .5 \\ +1, & r > .5 \end{cases} \tag{24}$$

is standardized double spike.

(b) Arc sine: If r is uniform between 0 and 1, then

$$AS = \sqrt{2} \sin [(r - 1/2)\pi] \tag{25}$$

is standardized arc sine.

Just as it is for the uniform population, the sample midrange is the maximum likelihood estimator of the location parameter (population mean) μ for the double spike and arc sine populations. Replacing the subscript U by DS and then by AS in Equation (1), we have

$$\hat{\mu}_{DS} = (x_1 + x_n)/2 \tag{26}$$

$$\hat{\mu}_{AS} = (x_1 + x_n)/2 \tag{27}$$

The sample range (or semirange) is a sufficient statistic for the scale parameter (population standard deviation) σ for the double spike and arc sine populations, just as it is for the uniform populations, but division by different constants is required to obtain the

maximum likelihood estimators, which are given by

$$\hat{\sigma}_{DS} = (x_n - x_1)/2 \tag{28}$$

$$\hat{\sigma}_{AS} = (x_n - x_1)/2\sqrt{2} \tag{29}$$

The canonical scale factors for the double spike and arc sine populations are $F_{DS} = 1.00000$ and $F_{AS} = 1.40986$, respectively.

Generation of random numbers and iterative estimation of location and scale parameters will be discussed in Appendix B for symmetric beta populations and in Appendix C for Student t populations.

The debiasing factor for the maximum likelihood estimator $\hat{\sigma}_{DS}$ of the scale parameter of a double spike population from a sample of size n is $C_{DS} = 2^{n-1}/(2^{n-1}-1)$, since $\hat{\sigma}_{DS} = 0$ with probability $1/2^{n-1}$ and $\hat{\sigma}_{DS} = \sigma_{DS}$ (the true value) with probability $(2^{n-1}-1)/2^{n-1}$. Closed form expressions for the debiasing factors for the maximum likelihood estimators of the scale parameters of the arc sine population, the six symmetric beta populations, and the six Student t populations are not available. Therefore, the Monte Carlo sample results were used to obtain the approximation $C_{AS} \doteq 5000/\sum_1^{5000}\hat{\sigma}_{AS}$ for the arc sine population and analogous approximations for the other populations. The debiasing factors for samples of size n = 8(4)24 for all 14 populations (other than U, N and D) are shown in Table 13 of Appendix A.

In Phase III, as in Phases I and II, the values $\hat{\mu}_U$, $\hat{\mu}_N$, $\hat{\mu}_D$, $\hat{\sigma}_U$, $\hat{\sigma}_N$, $\hat{\sigma}_D$, K, Q, L_U, L_N, L_D, λ_1, λ_2 and λ_3 were computed for each sample by use of Equations (1)-(12), and the results were used to classify each sample as U, N or D, after which the adaptive robust estimates of

location and scale parameters were calculated. An exception was necessary in the case of samples from the standardized double spike population that consisted of all +1's or all -1's. For such a sample, $\hat{\sigma}_U = \hat{\sigma}_N = \hat{\sigma}_D = 0$ and both the numerator and the denominator of the expression for Q in Equation (8) are zero, so K, Q, L_U, L_N, L_D, λ_1, λ_2 and λ_3 were not calculated and the sample was classified as U. The results of Phase III of the Monte Carlo study are shown in Tables 14-24 of Appendix A. Tables 14-18 are contingency tables showing the number of samples from each population [DS, AS, SB(1.5), SB(2.0), SB(2.5), SB(3.0), SB(3.5), SB(4.0), ST(16), ST(10), ST(8), ST(7), ST(6) and ST(5)] classified by each criterion as U, N and D, for n = 8(4)24, respectively. Criterion (c) was dropped because of its poor performance in Phases I and II. The mean square errors of $\hat{\mu}_T$, $\hat{\sigma}_T$, $\bar{\sigma}_T = C_T\hat{\sigma}_T$, $F_T\hat{\sigma}_T$ and $F_T\bar{\sigma}_T$, where the subscript T designates the true population from which the sample actually came, are shown in Table 19. The efficiencies of the adaptive robust estimates, relative to the maximum likelihood (or debiased maximum likelihood) estimates when the population from which each sample actually came is known, are shown in Tables 20-24 for $\hat{\mu}$, $\hat{\sigma}$, $\bar{\sigma}$, $F\hat{\sigma}$ and $F\bar{\sigma}$, respectively.

In Phase IV of the Monte Carlo study, critical values of the criteria [(a)(1), (a)(2), b(1), b(2), (d)(1), (d)(2), (d)(3) and (e)] for sample sizes n = 10(4)22, obtained by interpolation from the corresponding critical values for n = 8(4)24, were used to classify (as U, N or D) 5000 samples of each size from each of the seventeen populations considered in Phases I-III and the adaptive robust estimates were then calculated. The five-point Lagrangian interpolation formulas used to

obtain the critical values V (where V can be K_{L1}, K_{U1}, K_{L2}, K_{U2}, Q_{L1}, Q_{U1}, Q_{L2}, Q_{U2}, λ^*_{11}, λ^*_{21}, λ^*_{31}, λ^*_{12}, λ^*_{22}, λ^*_{32}, λ^*_{13}, λ^*_{23} or λ^*_{33} and the subscripts of V are values of the sample size n) are given by

$$V_{10} = 0.2734375V_8 + 1.09375V_{12} - 0.546875V_{16} + 0.21875V_{20} - 0.0390625V_{24} \quad (30)$$

$$V_{14} = -0.0390625V_8 + 0.46875V_{12} + 0.703125V_{16} - 0.15625V_{20} + 0.0234375V_{24} \quad (31)$$

$$V_{18} = 0.0234375V_8 - 0.15625V_{12} + 0.703125V_{16} + 0.46875V_{20} - 0.0390625V_{24} \quad (32)$$

$$V_{22} = -0.0390625V_8 + 0.21875V_{12} - 0.546875V_{16} + 1.09375V_{20} + 0.2734375V_{24} \quad (33)$$

In determining λ^*_{31} and λ^*_{32} for n = 14, 18, 22, use was made of the following four-point Lagrangian interpolation formulas, dropping the arbitrarily assigned values (1.0000) for n = 8 and using *only the values determined in the* usual manner for n = 12, 16, 20, 24:

$$V_{14} = 0.3125V_{12} + 0.9375V_{16} - 0.3125V_{20} + 0.0625V_{24} \quad (31')$$

$$V_{18} = -0.0625V_{12} + 0.5625V_{16} + 0.5625V_{20} - 0.0625V_{24} \quad (32')$$

$$V_{22} = 0.0625V_{12} - 0.3125V_{16} + 0.9375V_{20} + 0.3125V_{24} \quad (33')$$

Equations (30)-(33) were also used to obtain the debiasing factors for the scale parameters of the arc sine population, the six symmetric beta populations, and the six Student t populations for n = 10(4)22 from the corresponding values for n = 8(4)24. The critical values for the various criteria for samples of size n = 10(4)22 are shown in Table 25 of Appendix A, and the debiasing factors for all 14 populations other than U, N and D for n = 10(4)22 are shown in Table 26.

The results of Phase IV of the Monte Carlo study are shown in Tables 27-36 of Appendix A. Tables 27-30 are contingency tables showing the number of samples from each of the 17 populations [U, N, D, DS, AS, SB(1.5), SB(2.0), SB(2.5), SB(3.0), SB(3.5), SB(4.0), ST(16), ST(10), ST(8), ST(7), ST(6) and ST(5)] classified by each criterion [other than (c)] as U, N or D, for n = 10(4)22, respectively. The mean square errors of $\hat{\mu}_T$, $\hat{\sigma}_T$, $\overline{\sigma}_T = C_T\hat{\sigma}_T$, $F_T\hat{\sigma}_T$ and $F_T\overline{\sigma}_T$, where the subscript T designates the true population from which the sample actually came, are shown in Table 31. The efficiencies of the adaptive robust estimates, relative to the maximum likelihood (or debiased maximum likelihood) estimates when the population from which each sample actually came is known, are shown in Tables 32-36 for $\hat{\mu}$, $\hat{\sigma}$, $\overline{\sigma}$, $F\hat{\sigma}$ and $F\overline{\sigma}$ respectively.

SECTION IV

CONCLUSIONS AND RECOMMENDATIONS

The following conclusions may be drawn from the results of the Monte Carlo study tabulated in Appendix A:

(1) Table 1 shows that, with the exception of λ^*_{31} and λ^*_{32}, all of the critical values for the various classification criteria increase monotonically as the sample size n increases. This increase occurs because K, Q, λ_1, λ_2 and λ_3 are all biased downward, but the amount of the bias decreases as n increases. The exception occurs because only samples for which $L_N \geq L_U$ and $L_D \geq L_U$ are used in determining λ^*_{31} and λ^*_{32}. The value 1.0000 was assigned arbitrarily to λ^*_{31} and λ^*_{32} for n = 8, since both of these

inequalities were never simultaneously satisfied for any of the 15000 samples of size n = 8 drawn in Phase I, which made both λ_{31}^* and λ_{32}^* indeterminate. For $n \geq 12$, λ_{31}^* and λ_{32}^* are monotone decreasing.

(2) Tables 2-6 show that if the objective is to maximize the number (or proportion) of samples classified correctly, criterion (d)(3) is much better than criterion (c) and slightly better than all the others. Criterion (c) performs poorly because it is heavily biased in favor of the uniform population, especially for small samples. Criteria [other than (c)] based on the likelihood perform slightly better than those based on Q, which in turn perform slightly better than those based on K. For all of the criteria, the proportion of samples classified correctly increases monotonically as the sample size n increases. For criteria other than (c), the proportion of correct classifications increases from 47-49% for n = 8 to 70-74% for n = 24. The proportion of samples classified correctly in Phase II differs very little from that for Phase I, and the former actually exceeds the latter about half of the time. This leads to the conclusion that the differences observed are due to random sampling error, and not to any degradation of performance because the critical values determined in Phase I were used in Phase II.

(3) Table 7 shows that the debiased estimator $\bar{\sigma}$ has a smaller mean square error than $\hat{\sigma}$ for samples from the uniform population and on the average over all three populations, but a slightly larger one for samples from the normal and double exponential populations. Also, the debiased estimator $F\bar{\sigma}$ has a smaller mean square error than $F\hat{\sigma}$ for samples from the uniform population and on the average over all three populations, but a larger one for samples from the normal and double exponential

populations. The differences between the mean square errors in Phases I and II give an indication of the magnitude of the random sampling errors.

(4) Tables 8-12 show that if the objective is to maximize the efficiency of the estimates, criterion (c) still performs worse than any of the others, except in the case of $F\overline{\sigma}$ [Table 12], where it actually performs best for n = 8 and n = 12. The criteria based on the likelihood in general [and criterion (d)(3) in particular] do not enjoy the same superiority over those based on Q and K as when the objective is to maximize the proportion of samples classified correctly. For small samples (n = 8, 12, 16), criteria based on Q and K [especially criteria (b)(2) and (a)(2)] perform better than those based on the likelihood in the cases of $\hat{\mu}$, $\hat{\sigma}$ and $F\hat{\sigma}$, but not in the cases of $\overline{\sigma}$ and $F\overline{\sigma}$. As the sample size n increases, the efficiency of $\hat{\mu}$ for the best classification criterion increases from about 72% for n = 8 to about 79% for n = 24, that of $\hat{\sigma}$ (92-95%) and $F\hat{\sigma}$ (79-81%) remains almost constant, and that of $\overline{\sigma}$ and $F\overline{\sigma}$ decreases from about 102% for n = 8 to about 94% for n = 24 and from about 95% for n = 8 to about 86% for n = 24 respectively. As in Tables 2-6, the differences in the Phase I and Phase II results appear to reflect only random sampling error, with no degradation because the critical values determined in Phase I were used in Phase II.

(5) Table 13 shows that the debiasing factors for maximum likelihood estimators of the scale parameters decrease monotonically as the sample size increases, for all 14 populations studied in Phase III, over the range of sample sizes considered. For the Student t populations with small numbers of degrees of freedom, the debiasing factors are less than one for the larger sample sizes.

(6) Tables 14-18 show that all of the criteria considered tend to classify samples from platykurtic populations (those with α_4 considerably less than 3) as uniform, samples from mesokurtic populations (those with α_4 near 3) as normal, and samples from leptokurtic populations (those with α_4 considerably greater than 3) as double exponential, especially for the larger sample sizes. Since there is no such thing as a "correct" classification in Phase III, it is impossible to say, on the basis of these results, which criterion performs best.

(7) Table 19 shows that the debiased maximum likelihood estimates $\bar{\sigma}$ and $F\bar{\sigma}$ tend to have smaller mean square errors than the corresponding maximum likelihood estimates $\hat{\sigma}$ and $F\hat{\sigma}$, respectively, for samples from symmetric beta populations (including arc sine but not double spike), but sometimes have larger ones for Student t populations, especially for large numbers of degrees of freedom and small sample sizes.

(8) Tables 20-24 show that adaptive robust estimates using criteria based on the likelihood [especially (d)(2)] tend to have higher efficiency, relative to the maximum likelihood estimates if the population is known, than those based on K and Q, except in the cases of $\hat{\mu}$ for the Student t populations and $\hat{\sigma}$ for the symmetric beta populations (including double spike and arc sine). In the case of $\hat{\mu}$ for Student t populations, criteria based on K [especially (a)(2)] tend to have the highest efficiency, while in the case of $\hat{\sigma}$ for symmetric beta populations, criteria based on K [especially (a)(2)] or on Q [especially (b)(2)] tend to have the highest efficiency. For $\hat{\mu}$ and $F\hat{\sigma}$, the relative efficiency tends to increase with the sample size, but for $\hat{\sigma}$, $\bar{\sigma}$ and $F\bar{\sigma}$, it tends to decrease.

(9) Table 25 shows that interpolation in Table 1 by use of Equations (30)-(33) [or (31')-(33')] is reasonably smooth. The minor irregularities that occur are not sufficient to have an appreciable effect on the performance of the adaptive robust estimators for n = 10 (4) 22, which is not very sensitive to minor variations in the critical values.

(10) Table 26 shows that interpolation in Table 13 by use of Equations (30)-(33) is also reasonably smooth, with only minor irregularities incapable of having an appreciable effect on the performance of the debiased estimators $\bar{\sigma}$ and $F\bar{\sigma}$ for n = 10 (4) 22.

(11) Tables 27-30 confirm the conclusion [see (2) above] reached from Tables 2-6 concerning the slight superiority of criterion (d)(3) as measured by the number (or proportion) of samples from U, N and D classified correctly. They also confirm the conclusion [see (6) above] reached from Tables 14-18 concerning the tendency to classify samples from platykurtic, mesokurtic and leptokurtic populations as uniform, normal and double exponential, respectively.

(12) Table 31 confirms the conclusions [see (3) and (7) above] reached from Tables 7 and 19 concerning the magnitude of the mean square errors of $\hat{\sigma}$ and $\bar{\sigma}$ and those of $F\hat{\sigma}$ and $F\bar{\sigma}$ for the various populations included in the study.

(13) Tables 32-36 show that, averaged over all 17 populations and all 4 sample sizes considered, estimates based on criterion (d)(2) tend to have the highest efficiency. However, those based on criterion (a)(2) have the highest efficiency in the case of $\hat{\mu}$ for Student t populations, while those based on K and Q [criteria (a)(1), (a)(2),

(b)(1) and (b)(2)] tend to do well in the case of $\hat{\sigma}$ for symmetric beta populations, as do those based on criterion (d)(3) in the case of $\hat{\sigma}$ for samples of size 10 from all populations, especially Student t.

(14) Taken as a whole, the results show that adaptive robust estimates of location and scale parameters based on all the criteria [except (c)] studied have, over a broad range of populations from the uniform ($\alpha_4 = 1.8$) to the Student t with 5 d.f. ($\alpha_4 = 9$), quite high efficiency relative to the maximum likelihood estimates if the population is known. The relative efficiency of the adaptive robust estimates is low for the double spike ($\alpha_4 = 1$) and arc sine ($\alpha_4 = 1.5$) populations, not because their mean square errors are very high, but because those of the maximum likelihood estimates are very low (zero for samples of size $n \geq 16$ from the double spike population).

(15) The Monte Carlo results show one rather surprising phenomenon. Some of the relative efficiencies of the adaptive robust estimators are larger than 1 (100%). This is partially, but not completely, accounted for by the effects of bias and canonical scale factors. The maximum relative efficiency is 1.3453 for $\hat{\sigma}$, but only 1.3168 for $\bar{\sigma}$, 1.2349 for $F\hat{\sigma}$ and 1.1328 for $F\bar{\sigma}$. For $\hat{\mu}$, it never exceeds 1.0492. Another possible explanation is that if a sample from population A behaves more like a sample from population B, where A is any one of the 17 populations considered and B is U, N or D, it may actually be more efficient to use the estimator appropriate for population B, which would be the adaptive robust estimator.

If the population is known, it is recommended that the appropriate ML or debiased ML estimators of its location and scale parameters be used, but if nothing is known about the population other than that it is symmetric, it is recommended that adaptive robust estimators be used. Those based on criterion (c) should be avoided, but differences among the others are small.

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APPENDIX A

RESULTS OF MONTE CARLO STUDY

Table 1

Critical Values of Criteria for Classification as U, N or D

(Determined in Phase I and Used Also in Phases II and III)

Criterion	Critical Values	n = 8	n = 12	n = 16	n = 20	n = 24
(a)(1)	K_{L1}	1.9966	2.0785	2.1236	2.1427	2.1495
	K_{U1}	2.3223	2.6278	2.7994	2.9123	3.0289
(a)(2)	K_{L2}	1.8952	2.0122	2.0796	2.1098	2.1342
	K_{U2}	2.4816	2.7103	2.8374	2.9277	3.0341
(b)(1)	Q_{L1}	1.8487	1.9807	2.0620	2.1207	2.1600
	Q_{U1}	2.0340	2.2914	2.4750	2.6139	2.7393
(b)(2)	Q_{L2}	1.7803	1.9414	2.0367	2.1015	2.1503
	Q_{U2}	2.1063	2.3253	2.4879	2.6191	2.7416
(c)	None--classify as U, N or D, whichever gives greatest likelihood					
(d)(1)	λ^*_{11}	0.7276	0.7844	0.8202	0.8470	0.8627
	λ^*_{21}	0.8150	0.8542	0.8865	0.8900	0.9066
	λ^*_{31}	1.0000	1.0700	1.0260	1.0143	1.0073
(d)(2)	λ^*_{12}	0.7126	0.7734	0.8124	0.8403	0.8583
	λ^*_{22}	0.8293	0.8821	0.9060	0.9224	0.9272
	λ^*_{32}	1.0000	1.0695	1.0270	1.0141	1.0091
(d)(3), (e)	λ^*_{13}	0.7368	0.7894	0.8238	0.8488	0.8640
	λ^*_{23}	0.7069	0.7685	0.8040	0.8272	0.8518
	λ^*_{33}	0.9632	0.9737	0.9801	0.9834	0.9860

Table 2

Contingency Tables--Classification vs. True Population by Criteria (n = 8)

Crite-rion			Phase I Classified as U	N	D	Sums	Phase II Classified as U	N	D	Sums
(a)(1)	Number of	U	3091	881	1028	5000	3060	903	1037	5000
		N	1909	1067	2024	5000	1899	1067	2034	5000
	Samples from	D	1127	897	2976	5000	1110	865	3025	5000
		Sums	6127	2845	6028	7134*	6069	2835	6096	7152*
(a)(2)	Number of	U	2641	1585	774	5000	2661	1554	785	5000
		N	1517	1857	1626	5000	1515	1868	1617	5000
	Samples from	D	842	1558	2600	5000	836	1521	2643	5000
		Sums	5000	5000	5000	7098*	5012	4943	5045	7172*
(b)(1)	Number of	U	3122	936	942	5000	3155	873	972	5000
		N	1878	1093	2029	5000	1919	1105	1976	5000
	Samples from	D	1127	902	2971	5000	1128	835	3037	5000
		Sums	6127	2931	5942	7186*	6202	2813	5985	7297*
(b)(2)	Number of	U	2688	1604	708	5000	2695	1571	734	5000
		N	1473	1860	1667	5000	1474	1908	1618	5000
	Samples from	D	839	1536	2625	5000	844	1493	2663	5000
		Sums	5000	5000	5000	7173*	5013	4972	5015	7266*
(c)	Number of	U	4961	0	39	5000	4960	0	40	5000
		N	4858	0	142	5000	4854	0	146	5000
	Samples from	D	4495	0	505	5000	4491	0	509	5000
		Sums	14314	0	686	5466*	14305	0	695	5469*
(d)(1)	Number of	U	3088	1533	379	5000	3104	1505	391	5000
		N	1828	2180	992	5000	1790	2222	988	5000
	Samples from	D	1264	1810	1926	5000	1251	1775	1974	5000
		Sums	6180	5523	3297	7194*	6145	5502	3353	7300*
(d)(2)	Number of	U	2753	1895	352	5000	2768	1887	345	5000
		N	1583	2489	928	5000	1556	2532	912	5000
	Samples from	D	1157	2024	1819	5000	1134	1993	1873	5000
		Sums	5493	6408	3099	7061*	5458	6412	3130	7173*
(d)(3)	Number of	U	3055	1317	628	5000	3075	1305	620	5000
		N	1733	2007	1260	5000	1765	2013	1222	5000
	Samples from	D	1070	1649	2281	5000	1073	1639	2288	5000
		Sums	5858	4973	4169	7343*	5913	4957	4130	7376*
(e)	Number of	U	3016	991	993	5000	3041	962	997	5000
		N	1698	1381	1921	5000	1736	1440	1824	5000
	Samples from	D	1029	1081	2890	5000	1043	1071	2886	5000
		Sums	5743	3453	5804	7287*	5820	3473	5707	7367*

* Numbers in this position are diagonal sums (the numbers of samples classified correctly)

Table 3

Contingency Tables--Classification vs. True Population by Criteria (n = 12)

Criterion			Phase I Classified as U	N	D	Sums	Phase II Classified as U	N	D	Sums
(a)(1)	Number of	U	3463	1100	437	5000	3374	1185	441	5000
		N	1535	1676	1789	5000	1536	1710	1754	5000
	Samples from	D	630	1161	3209	5000	603	1212	3185	5000
		Sums	5628	3937	5435	8348*	5513	4107	5380	8269*
(a)(2)	Number of	U	3203	1423	374	5000	3095	1538	367	5000
		N	1284	2110	1606	5000	1307	2162	1531	5000
	Samples from	D	513	1467	3020	5000	486	1484	3030	5000
		Sums	5000	5000	5000	8333*	4888	5184	4928	8287*
(b)(1)	Number of	U	3507	1113	380	5000	3458	1144	398	5000
		N	1492	1739	1769	5000	1505	1720	1775	5000
	Samples from	D	613	1156	3231	5000	613	1203	3184	5000
		Sums	5612	4008	5380	8477*	5576	4067	5357	8362*
(b)(2)	Number of	U	3258	1428	314	5000	3200	1468	332	5000
		N	1249	2134	1617	5000	1289	2121	1590	5000
	Samples from	D	493	1438	3069	5000	500	1452	3048	5000
		Sums	5000	5000	5000	8461*	4989	5041	4970	8369*
(c)	Number of	U	4964	0	36	5000	4954	0	46	5000
		N	4563	8	429	5000	4554	9	437	5000
	Samples from	D	3467	9	1524	5000	3426	14	1560	5000
		Sums	12994	17	1989	6496*	12934	23	2043	6523*
(d)(1)	Number of	U	3532	1294	174	5000	3417	1377	206	5000
		N	1424	2585	991	5000	1428	2596	976	5000
	Samples from	D	725	1895	2380	5000	726	1868	2406	5000
		Sums	5681	5774	3545	8497*	5571	5841	3588	8419*
(d)(2)	Number of	U	3315	1544	141	5000	3212	1624	164	5000
		N	1321	2755	904	5000	1346	2776	878	5000
	Samples from	D	725	2011	2264	5000	728	1979	2293	5000
		Sums	5361	6330	3309	8354*	5286	6379	3335	8281*
(d)(3)	Number of	U	3514	1189	297	5000	3428	1257	315	5000
		N	1396	2427	1177	5000	1370	2468	1162	5000
	Samples from	D	611	1740	2649	5000	631	1695	2674	5000
		Sums	5521	5356	4123	8590*	5429	5420	4151	8570*
(e)	Number of	U	3502	1003	495	5000	3415	1045	540	5000
		N	1384	1908	1708	5000	1363	1946	1691	5000
	Samples from	D	604	1221	3175	5000	621	1180	3199	5000
		Sums	5490	4132	5378	8585*	5399	4171	5430	8560*

* Numbers in this position are diagonal sums (the numbers of samples classified correctly)

Table 4

Contingency Tables--Classification vs. True Population by Criteria (n = 16)

Criterion			Phase I Classified as U	N	D	Sums	Phase II Classified as U	N	D	Sums
(a)(1)	Number of	U	3714	1106	180	5000	3715	1098	187	5000
		N	1286	2108	1606	5000	1293	2109	1598	5000
	Samples from	D	347	1259	3394	5000	338	1264	3398	5000
		Sums	5347	4473	5180	9216*	5346	4471	5183	9222*
(a)(2)	Number of	U	3554	1283	163	5000	3537	1301	162	5000
		N	1153	2324	1523	5000	1134	2345	1521	5000
	Samples from	D	293	1393	3314	5000	289	1380	3331	5000
		Sums	5000	5000	5000	9192*	4960	5026	5014	9213*
(b)(1)	Number of	U	3801	1071	128	5000	3752	1122	126	5000
		N	1199	2192	1609	5000	1183	2179	1638	5000
	Samples from	D	322	1287	3391	5000	317	1259	3424	5000
		Sums	5322	4550	5128	9384*	5252	4560	5188	9355*
(b)(2)	Number of	U	3656	1223	121	5000	3618	1273	109	5000
		N	1071	2389	1540	5000	1036	2390	1574	5000
	Samples from	D	273	1388	3339	5000	274	1354	3372	5000
		Sums	5000	5000	5000	9384*	4928	5017	5055	9380*
(c)	Number of	U	4964	11	25	5000	4968	7	25	5000
		N	4143	204	653	5000	4144	210	646	5000
	Samples from	D	2446	198	2356	5000	2447	210	2343	5000
		Sums	11553	413	3034	7524*	11559	427	3014	7521*
(d)(1)	Number of	U	3843	1061	96	5000	3822	1084	94	5000
		N	1131	3002	867	5000	1112	2980	908	5000
	Samples from	D	429	1928	2643	5000	435	1863	2702	5000
		Sums	5403	5991	3606	9488*	5369	5927	3704	9504*
(d)(2)	Number of	U	3693	1230	77	5000	3697	1227	76	5000
		N	1053	3129	818	5000	1043	3098	859	5000
	Samples from	D	422	1984	2594	5000	434	1919	2647	5000
		Sums	5168	6343	3489	9416*	5174	6244	3582	9442*
(d)(3)	Number of	U	3842	1000	158	5000	3819	1022	159	5000
		N	1118	2718	1164	5000	1114	2742	1144	5000
	Samples from	D	374	1587	3039	5000	363	1573	3064	5000
		Sums	5334	5305	4361	9599*	5296	5337	4367	9625*
(e)	Number of	U	3835	902	263	5000	3806	935	259	5000
		N	1111	2333	1556	5000	1105	2328	1567	5000
	Samples from	D	370	1240	3390	5000	358	1198	3444	5000
		Sums	5316	4475	5209	9558*	5269	4461	5270	9578*

* Numbers in this position are diagonal sums (the numbers of samples classified correctly)

Table 5

Contingency Tables--Classification vs. True Population by Criteria (n = 20)

Criterion			Phase I Classified as U	N	D	Sums	Phase II Classified as U	N	D	Sums
(a)(1)	Number of	U	3943	994	63	5000	3967	961	72	5000
		N	1057	2468	1475	5000	1129	2402	1469	5000
	Samples from	D	245	1230	3525	5000	203	1175	3622	5000
		Sums	5245	4692	5063	9936*	5299	4538	5163	9991*
(a)(2)	Number of	U	3825	1114	61	5000	3829	1103	68	5000
		N	961	2599	1440	5000	1014	2551	1435	5000
	Samples from	D	214	1287	3499	5000	172	1239	3589	5000
		Sums	5000	5000	5000	9923*	5015	4893	5092	9969*
(b)(1)	Number of	U	4080	877	43	5000	4063	898	39	5000
		N	920	2592	1488	5000	1032	2504	1464	5000
	Samples from	D	232	1256	3512	5000	194	1204	3602	5000
		Sums	5232	4725	5043	10184*	5289	4606	5105	10169*
(b)(2)	Number of	U	3962	995	43	5000	3966	997	37	5000
		N	834	2701	1465	5000	928	2627	1445	5000
	Samples from	D	204	1304	3492	5000	174	1241	3585	5000
		Sums	5000	5000	5000	10155*	5068	4865	5067	10178*
(c)	Number of	U	4967	14	19	5000	4967	20	13	5000
		N	3625	620	755	5000	3634	608	758	5000
	Samples from	D	1723	458	2819	5000	1619	463	2918	5000
		Sums	10315	1092	3593	8406*	10220	1091	3689	8493*
(d)(1)	Number of	U	4105	851	44	5000	4113	845	42	5000
		N	865	3302	833	5000	956	3198	846	5000
	Samples from	D	285	1801	2914	5000	244	1727	3029	5000
		Sums	5255	5954	3791	10321*	5313	5770	3917	10340*
(d)(2)	Number of	U	3975	991	34	5000	4017	953	30	5000
		N	818	3387	795	5000	878	3300	822	5000
	Samples from	D	316	1820	2864	5000	271	1753	2976	5000
		Sums	5109	6198	3693	10226*	5166	6006	3828	10293*
(d)(3)	Number of	U	4114	815	71	5000	4127	804	69	5000
		N	874	2970	1156	5000	961	2898	1141	5000
	Samples from	D	266	1396	3338	5000	226	1342	3432	5000
		Sums	5254	5181	4565	10422*	5314	5044	4642	10457*
(e)	Number of	U	4110	775	115	5000	4120	753	127	5000
		N	868	2724	1408	5000	960	2659	1381	5000
	Samples from	D	265	1157	3578	5000	221	1141	3638	5000
		Sums	5243	4656	5101	10412*	5301	4553	5146	10417*

* Numbers in this position are diagonal sums (the number of samples classified correctly)

Table 6

Contingency Tables--Classification vs. True Population by Criteria (n = 24)

Crite-rion			Phase I Classified as				Phase II Classified as			
			U	N	D	Sums	U	N	D	Sums
(a)(1)	Number of	U	4105	869	26	5000	4053	918	29	5000
		N	895	2732	1373	5000	916	2732	1352	5000
	Samples from	D	111	1262	3627	5000	98	1230	3672	5000
		Sums	5111	4863	5026	10464*	5067	4880	5053	10457*
(a)(2)	Number of	U	4051	923	26	5000	4005	967	28	5000
		N	851	2791	1358	5000	869	2791	1340	5000
	Samples from	D	98	1286	3616	5000	94	1244	3662	5000
		Sums	5000	5000	5000	10458*	4968	5002	5030	10458*
(b)(1)	Number of	U	4236	752	12	5000	4187	798	15	5000
		N	764	2862	1374	5000	817	2821	1362	5000
	Samples from	D	101	1273	3626	5000	86	1252	3662	5000
		Sums	5101	4887	5012	10724*	5090	4871	5039	10670*
(b)(2)	Number of	U	4186	803	11	5000	4141	844	15	5000
		N	719	2911	1370	5000	785	2862	1353	5000
	Samples from	D	95	1286	3619	5000	80	1262	3658	5000
		Sums	5000	5000	5000	10716*	5006	4968	5026	10661*
(c)	Number of	U	4970	24	6	5000	4968	23	9	5000
		N	3109	1104	787	5000	3101	1128	771	5000
	Samples from	D	1098	702	3200	5000	1078	702	3220	5000
		Sums	9177	1830	3993	9274*	9147	1853	4000	9316*
(d)(1)	Number of	U	4266	712	22	5000	4211	759	30	5000
		N	720	3467	813	5000	716	3482	802	5000
	Samples from	D	138	1635	3227	5000	119	1608	3273	5000
		Sums	5124	5814	4062	10960*	5046	5849	4105	10966*
(d)(2)	Numbers of	U	4216	767	17	5000	4156	819	25	5000
		N	681	3538	781	5000	677	3565	758	5000
	Samples from	D	143	1678	3179	5000	128	1657	3215	5000
		Sums	5040	5983	3977	10933*	4961	6041	3998	10936*
(d)(3)	Number of	U	4266	692	42	5000	4214	741	45	5000
		N	727	3150	1123	5000	721	3174	1105	5000
	Samples from	D	125	1264	3611	5000	105	1241	3654	5000
		Sums	5118	5106	4776	11027*	5040	5156	4804	11042*
(e)	Number of	U	4265	669	66	5000	4212	707	81	5000
		N	727	3024	1249	5000	720	3035	1245	5000
	Samples from	D	124	1138	3738	5000	104	1112	3784	5000
		Sums	5116	4831	5053	11027*	5036	4854	5110	11031*

* Numbers in this position are diagonal sums (the numbers of samples classified correctly)

Table 7

Mean Square Errors of Parameter Estimates if Population is Known

Estimate	Population	Phase	MSE(n=8)	MSE(n=12)	MSE(n=16)	MSE(n=20)	MSE(n=24)
$\hat{\mu}$	U	I	.069054	.033113	.019490	.012646	.009254
		II	.068616	.032782	.018859	.012844	.009102
	N	I	.123065	.084018	.061762	.049432	.041521
		II	.122679	.086460	.061833	.049238	.041767
	D	I	.091123	.059292	.040708	.033023	.026357
		II	.092905	.057610	.042764	.033049	.027138
	Avg.	I	.094414	.058808	.040654	.031701	.025711
		II	.094733	.058951	.041152	.031711	.026002
$\hat{\sigma}$	U	I	.067185	.033359	.019543	.013061	.009387
		II	.068304	.033136	.019486	.012860	.009051
	N	I	.069657	.044004	.032708	.026163	.022315
		II	.069162	.044913	.033501	.026587	.021940
	D	I	.121718	.086484	.062850	.050400	.041594
		II	.123134	.081256	.063581	.049865	.041672
	Avg.	I	.086187	.054616	.038367	.029875	.024432
		II	.086867	.053102	.038856	.029771	.024221
$F\hat{\sigma}$	U	I	.181905	.090319	.052913	.035362	.025415
		II	.184934	.089716	.052758	.034820	.024505
	N	I	.267585	.169040	.125645	.100505	.085724
		II	.265682	.172531	.128693	.102133	.084280
	D	I	.546186	.388081	.282027	.226161	.186645
		II	.552541	.364622	.285308	.223761	.186993
	Avg.	I	.331892	.215813	.153528	.120676	.099261
		II	.334386	.208956	.155586	.120238	.098593
$\bar{\sigma}$	U	I	.029276	.013098	.007479	.004902	.003360
		II	.028841	.013312	.007248	.004691	.003286
	N	I	.073322	.045791	.033350	.026863	.022432
		II	.073047	.046188	.033408	.027211	.022423
	D	I	.135071	.093458	.066026	.052916	.042905
		II	.136590	.086380	.066893	.051937	.042798
	Avg.	I	.079223	.050782	.035619	.028227	.022899
		II	.079493	.048627	.036183	.027946	.022836
$F\bar{\sigma}$	U	I	.079264	.035463	.020250	.013272	.009096
		II	.078087	.036041	.019624	.012701	.008898
	N	I	.281662	.175902	.128113	.103192	.086173
		II	.280607	.177429	.132175	.104531	.086136
	D	I	.606109	.419375	.296282	.237450	.192531
		II	.612922	.387617	.300172	.233059	.192049
	Avg.	I	.322345	.210247	.148215	.117971	.095933
		II	.323872	.200363	.150657	.116763	.095694

Canonical Scale Factors: $F_U = 1.64545$, $F_N = 1.95996$, $F_D = 2.11833$

Table 8

Efficiencies of Adaptive Robust Estimates of Location Parameter

(Relative to Maximum Likelihood Estimate if Population is Known)

Sample Size n	Criterion	Phase I Samples from U	N	D	Avg.	Phase II Samples from U	N	D	Avg.
8	(a)(1)	.4672	.8304	.8073	.6928	.4689	.8264	.8079	.6935
	(a)(2)	.4786	.8656	.8352	.7161	.4833	.8662	.8371	.7203
	(b)(1)	.5251	.8081	.7722	.7049	.5253	.7982	.7523	.6969
	(b)(2)	.5287	.8556	.7875	.7260	.5299	.8489	.7845	.7242
	(c)	.9515	.6362	.2920	.4900	.9492	.6205	.2872	.4789
	(d)(1)	.6147	.8222	.6007	.6846	.6219	.8230	.6014	.6867
	(d)(2)	.6102	.8284	.5831	.6776	.6231	.8264	.5880	.6822
	(d)(3)	.5257	.8265	.7465	.7040	.5337	.8212	.7367	.7034
	(e)	.4898	.7996	.7776	.6874	.4961	.8024	.7556	.6862
12	(a)(1)	.4439	.8166	.8440	.7121	.4437	.8286	.8111	.7095
	(a)(2)	.4400	.8379	.8447	.7180	.4430	.8490	.8205	.7188
	(b)(1)	.5279	.7978	.7933	.7267	.5406	.7996	.7594	.7230
	(b)(2)	.5203	.8225	.8140	.7393	.5260	.8247	.7703	.7309
	(c)	.9706	.5258	.2838	.4380	.9513	.5369	.2804	.4411
	(d)(1)	.5864	.7945	.6507	.6964	.5940	.8011	.6162	.6892
	(d)(2)	.5855	.7917	.6087	.6783	.5932	.8048	.5785	.6743
	(d)(3)	.5105	.8036	.7574	.7123	.5307	.8110	.7125	.7096
	(e)	.4795	.7813	.7769	.6976	.4950	.7887	.7338	.6953
16	(a)(1)	.4209	.7991	.8637	.7143	.4094	.8181	.8629	.7211
	(a)(2)	.4106	.8209	.8688	.7193	.4021	.8377	.8637	.7252
	(b)(1)	.5312	.7881	.8292	.7430	.5167	.7980	.8275	.7453
	(b)(2)	.5160	.8084	.8435	.7508	.5012	.8090	.8341	.7467
	(c)	.9583	.4901	.3303	.4523	.9484	.4915	.3272	.4467
	(d)(1)	.5688	.7950	.6987	.7165	.5494	.8100	.6966	.7176
	(d)(2)	.5630	.8088	.6838	.7153	.5489	.8057	.6806	.7098
	(d)(3)	.5207	.7974	.7790	.7297	.5053	.8135	.7833	.7352
	(e)	.4959	.7763	.7997	.7184	.4702	.7971	.8082	.7237
20	(a)(1)	.4131	.8191	.8643	.7362	.4119	.7952	.8665	.7249
	(a)(2)	.3990	.8307	.8732	.7371	.3930	.8099	.8719	.7241
	(b)(1)	.5245	.8165	.8262	.7631	.5264	.7827	.8374	.7504
	(b)(2)	.4958	.8253	.8348	.7611	.5107	.7979	.8488	.7562
	(c)	.9268	.4945	.3824	.4756	.9586	.4748	.3844	.4685
	(d)(1)	.5526	.8216	.7618	.7524	.5769	.8058	.7579	.7492
	(d)(2)	.5328	.8261	.7192	.7344	.5640	.8134	.7096	.7324
	(d)(3)	.5308	.8084	.8102	.7564	.5439	.7839	.8103	.7478
	(e)	.5185	.7947	.8297	.7524	.5169	.7718	.8229	.7386
24	(a)(1)	.4144	.8328	.8925	.7583	.3985	.8205	.8929	.7491
	(a)(2)	.4061	.8360	.8953	.7570	.3930	.8248	.8950	.7492
	(b)(1)	.5303	.8267	.8633	.7854	.5125	.8153	.8730	.7795
	(b)(2)	.5203	.8374	.8659	.7886	.4991	.8242	.8735	.7802
	(c)	.9457	.5045	.4714	.5212	.9530	.5156	.4670	.5247
	(d)(1)	.5666	.8301	.8352	.7878	.5357	.8461	.8166	.7833
	(d)(2)	.5548	.8345	.8159	.7811	.5305	.8461	.7900	.7733
	(d)(3)	.5392	.8213	.8673	.7862	.5197	.8325	.8889	.7943
	(e)	.5309	.8161	.8757	.7838	.5030	.8290	.8976	.7903

Table 9

Efficiencies of Adaptive Robust Estimates of Scale Parameter

(Relative to Maximum Likelihood Estimate if Population is Known)

Sample Size n	Criterion	Phase I Samples from U	N	D	Avg.	Phase II Samples from U	N	D	Avg.
8	(a)(1)	1.1468	.8169	.8939	.9234	1.1544	.8141	.8972	.9262
	(a)(2)	1.1915	.8386	.9038	.9432	1.1905	.8365	.9080	.9454
	(b)(1)	1.1390	.8115	.8862	.9163	1.1389	.8133	.8957	.9225
	(b)(2)	1.1785	.8332	.8976	.9361	1.1794	.8338	.9080	.9426
	(c)	.9925	.7852	.8569	.8663	.9925	.7829	.8716	.8732
	(d)(1)	1.1058	.8226	.8963	.9194	1.1012	.8233	.9060	.9243
	(d)(2)	1.1517	.8379	.9044	.9366	1.1487	.8380	.9166	.9431
	(d)(3)	1.1110	.8244	.8934	.9195	1.1060	.8251	.9011	.9234
	(e)	1.1353	.8213	.8905	.9212	1.1361	.8252	.8997	.9281
12	(a)(1)	1.0387	.8774	.9324	.9361	1.0370	.8667	.9172	.9242
	(a)(2)	1.0538	.8880	.9348	.9431	1.0501	.8782	.9199	.9314
	(b)(1)	1.0236	.8725	.9307	.9312	1.0187	.8684	.9114	.9187
	(b)(2)	1.0353	.8808	.9346	.9378	1.0307	.8769	.9145	.9250
	(c)	.9866	.8560	.9122	.9101	.9833	.8549	.8990	.9020
	(d)(1)	.9852	.8886	.9337	.9309	.9866	.8823	.9094	.9164
	(d)(2)	1.0039	.8928	.9381	.9378	1.0059	.8886	.9144	.9243
	(d)(3)	.9898	.8859	.9331	.9306	.9875	.8806	.9065	.9145
	(e)	1.0102	.8773	.9327	.9315	1.0097	.8714	.9076	.9161
16	(a)(1)	.9474	.9169	.9402	.9347	.9513	.9176	.9583	.9451
	(a)(2)	.9497	.9210	.9394	.9358	.9537	.9238	.9580	.9472
	(b)(1)	.9245	.9200	.9357	.9293	.9248	.9203	.9617	.9432
	(b)(2)	.9283	.9228	.9361	.9310	.9256	.9275	.9608	.9450
	(c)	.9798	.8588	.9312	.9169	.9833	.8561	.9592	.9308
	(d)(1)	.9027	.9433	.9252	.9263	.9029	.9397	.9506	.9392
	(d)(2)	.9094	.9449	.9260	.9284	.9083	.9411	.9511	.9408
	(d)(3)	.9067	.9416	.9267	.9274	.9072	.9372	.9515	.9397
	(e)	.9228	.9333	.9310	.9302	.9223	.9303	.9525	.9409
20	(a)(1)	.8800	.9250	.9470	.9302	.8886	.9249	.9566	.9367
	(a)(2)	.8796	.9308	.9466	.9316	.8869	.9277	.9563	.9371
	(b)(1)	.8561	.9309	.9428	.9257	.8577	.9341	.9516	.9317
	(b)(2)	.8549	.9330	.9433	.9263	.8563	.9352	.9520	.9320
	(c)	.9769	.8315	.9361	.9082	.9735	.8153	.9575	.9123
	(d)(1)	.8422	.9708	.9360	.9306	.8441	.9663	.9400	.9323
	(d)(2)	.8407	.9709	.9356	.9302	.8430	.9662	.9395	.9318
	(d)(3)	.8461	.9627	.9382	.9304	.8483	.9597	.9440	.9334
	(e)	.8563	.9605	.9403	.9327	.8587	.9519	.9468	.9345
24	(a)(1)	.8320	.9271	.9545	.9287	.8158	.9222	.9403	.9174
	(a)(2)	.8308	.9292	.9531	.9283	.8120	.9252	.9403	.9177
	(b)(1)	.8091	.9456	.9547	.9305	.7926	.9204	.9418	.9140
	(b)(2)	.8060	.9466	.9544	.9301	.7888	.9228	.9415	.9139
	(c)	.9685	.7867	.9592	.9002	.9656	.7826	.9499	.8940
	(d)(1)	.7995	.9738	.9424	.9302	.7813	.9727	.9323	.9217
	(d)(2)	.7967	.9755	.9418	.9299	.7778	.9738	.9323	.9214
	(d)(3)	.8035	.9700	.9460	.9319	.7848	.9596	.9341	.9197
	(e)	.8098	.9657	.9473	.9324	.7928	.9466	.9354	.9181

Table 10

Efficiencies of Adaptive Robust Estimates of Canonical Scale Parameter

(Relative to Maximum Likelihood Estimate if Population is Known)

Sample Size n	Criterion	Phase I Samples from U	N	D	Avg.	Phase II Samples from U	N	D	Avg.
8	(a)(1)	1.0724	.6174	.7588	.7527	1.1194	.6114	.7627	.7575
	(a)(2)	1.1727	.6654	.7831	.7935	1.2159	.6601	.7871	.7983
	(b)(1)	1.1347	.6152	.7501	.7524	1.1781	.6132	.7585	.7607
	(b)(2)	1.2253	.6631	.7758	.7927	1.2715	.6610	.7864	.8025
	(c)	1.0023	.5116	.6078	.6211	1.0020	.5099	.6175	.6268
	(d)(1)	1.2419	.6477	.7429	.7690	1.2601	.6454	.7540	.7769
	(d)(2)	1.2907	.6784	.7574	.7924	1.3168	.6745	.7726	.8028
	(d)(3)	1.2003	.6509	.7534	.7733	1.2293	.6466	.7602	.7787
	(e)	1.1326	.6375	.7576	.7652	1.1804	.6377	.7657	.7747
12	(a)(1)	1.0723	.6252	.8029	.7726	1.0784	.6248	.7828	.7597
	(a)(2)	1.1091	.6573	.8143	.7942	1.1194	.6572	.7949	.7823
	(b)(1)	1.1440	.6254	.8022	.7773	1.1321	.6276	.7743	.7598
	(b)(2)	1.1779	.6541	.8163	.7988	1.1684	.6558	.7874	.7807
	(c)	1.0037	.5505	.6767	.6671	1.0043	.5569	.6283	.6400
	(d)(1)	1.1832	.6679	.7859	.7865	1.1821	.6678	.7553	.7673
	(d)(2)	1.1986	.6838	.7959	.7991	1.2092	.6851	.7642	.7805
	(d)(3)	1.1452	.6618	.7962	.7879	1.1398	.6647	.7589	.7657
	(e)	1.0901	.6387	.8042	.7800	1.0730	.6404	.7688	.7577
16	(a)(1)	1.0354	.6466	.8169	.7798	1.0284	.6531	.8363	.7918
	(a)(2)	1.0380	.6654	.8214	.7898	1.0421	.6720	.8410	.8028
	(b)(1)	1.1259	.6554	.8133	.7867	1.1297	.6589	.8444	.8049
	(b)(2)	1.1312	.6698	.8190	.7959	1.1360	.6805	.8479	.8159
	(c)	1.0022	.5977	.6794	.6792	1.0015	.6167	.7058	.7013
	(d)(1)	1.1225	.7119	.7768	.7850	1.1296	.7107	.8023	.8001
	(d)(2)	1.1358	.7255	.7792	.7918	1.1372	.7279	.8039	.8074
	(d)(3)	1.1046	.7004	.7881	.7872	1.1051	.6972	.8158	.8019
	(e)	1.0667	.6758	.8011	.7839	1.0663	.6739	.8262	.7968
20	(a)(1)	1.0094	.6716	.8218	.7872	1.0200	.6592	.8431	.7937
	(a)(2)	1.0095	.6813	.8259	.7933	1.0236	.6710	.8471	.8009
	(b)(1)	1.0763	.6738	.8191	.7902	1.0807	.6690	.8370	.7977
	(b)(2)	1.0799	.6813	.8246	.7965	1.0900	.6788	.8407	.8042
	(c)	1.0002	.6563	.7185	.7193	1.0004	.6518	.7346	.7271
	(d)(1)	1.0956	.7542	.7869	.7992	1.0807	.7385	.8003	.8014
	(d)(2)	1.1019	.7570	.7860	.7999	1.0925	.7469	.7987	.8038
	(d)(3)	1.0818	.7255	.8020	.7988	1.0662	.7129	.8179	.8025
	(e)	1.0688	.7099	.8113	.7984	1.0298	.6918	.8276	.7983
24	(a)(1)	.9899	.6856	.8449	.8013	.9710	.6670	.8352	.7877
	(a)(2)	.9811	.6917	.8459	.8038	.9679	.6717	.8351	.7894
	(b)(1)	1.0573	.6957	.8479	.8105	1.0520	.6584	.8393	.7906
	(b)(2)	1.0577	.7002	.8489	.8129	1.0562	.6636	.8398	.7932
	(c)	.9967	.6965	.7616	.7565	.9961	.6814	.7576	.7486
	(d)(1)	1.0600	.7670	.8102	.8134	1.0699	.7633	.8085	.8112
	(d)(2)	1.0637	.7752	.8085	.8151	1.0731	.7745	.8070	.8140
	(d)(3)	1.0505	.7407	.8290	.8157	1.0625	.7241	.8238	.8071
	(e)	1.0378	.7270	.8356	.8142	1.0323	.6997	.8302	.8007

Table 11

Efficiencies of Debiased Adaptive Robust Estimates of Scale Parameter

(Relative to Debiased Maximum Likelihood Estimate if Population is Known)

Sample Size n	Criterion	Phase I Samples from U	N	D	Avg.	Phase II Samples from U	N	D	Avg.
8	(a)(1)	.9266	.9427	1.0295	.9879	.9095	.9475	1.0377	.9919
	(a)(2)	.9250	.9723	1.0361	1.0010	.9055	.9745	1.0437	1.0034
	(b)(1)	.9473	.9531	1 0303	.9947	.9317	.9602	1.0378	.9993
	(b)(2)	.9396	.9757	1.0364	1.0044	.9194	.9834	1.0469	1.0100
	(c)	.9816	.7910	.8199	.8274	.9827	.7897	.8339	.8349
	(d)(1)	.9421	.9963	1.0456	1.0163	.9174	1.0045	1.0547	1.0206
	(d)(2)	.9424	.9902	1.0412	1.0121	.9178	.9983	1.0488	1.0155
	(d)(3)	.9402	1.0010	1.0505	1.0202	.9162	1.0070	1.0604	1.0243
	(e)	.9420	.9760	1.0359	1.0046	.9263	.9874	1.0475	1.0126
12	(a)(1)	.8453	.9222	1.0152	.9691	.8350	.9090	1.0050	.9553
	(a)(2)	.8323	.9428	1.0178	.9758	.8232	.9328	1.0068	.9630
	(b)(1)	.8469	.9307	1.0153	.9721	.8465	.9234	1.0043	.9613
	(b)(2)	.8347	.9452	1.0173	.9766	.8375	.9426	1.0054	.9673
	(c)	.9616	.6489	.8135	.7653	.9530	.6504	.8403	.7768
	(d)(1)	.8161	.9802	1.0256	.9900	.8141	.9768	1.0059	.9757
	(d)(2)	.8189	.9762	1.0167	.9840	.8146	.9710	.9982	.9696
	(d)(3)	.8154	.9816	1.0276	.9914	.8153	.9802	1.0100	.9792
	(e)	.8297	.9550	1.0184	.9797	.8302	.9529	1.0016	.9677
16	(a)(1)	.7660	.8945	.9977	.9438	.7656	.8967	1.0166	.9552
	(a)(2)	.7535	.9060	.9975	.9462	.7514	.9156	1.0168	.9605
	(b)(1)	.7559	.9104	.9964	.9474	.7428	.9157	1.0205	.9616
	(b)(2)	.7502	.9228	.9968	.9511	.7335	.9311	1.0199	.9655
	(c)	.9472	.5769	.8747	.7568	.9566	.5737	.8902	.7607
	(d)(1)	.7376	.9736	.9944	.9645	.7261	.9744	1.0214	.9798
	(d)(2)	.7352	.9740	.9927	.9633	.7249	.9688	1.0189	.9765
	(d)(3)	.7387	.9659	.9965	.9634	.7280	.9704	1.0216	.9789
	(e)	.7543	.9452	.9951	.9579	.7424	.9502	1.0157	.9706
20	(a)(1)	.6953	.8580	.9963	.9258	.6924	.8673	1.0003	.9308
	(a)(2)	.6843	.8748	.9955	.9303	.6780	.8795	.9998	.9336
	(b)(1)	.6825	.8885	.9927	.9334	.6740	.8953	.9971	.9374
	(b)(2)	.6727	.8997	.9925	.9361	.6631	.9046	.9972	.9395
	(c)	.9469	.5448	.8909	.7436	.9333	.5417	.9149	.7484
	(d)(1)	.6650	.9622	.9974	.9586	.6630	.9619	.9958	.9579
	(d)(2)	.6580	.9677	.9926	.9566	.6560	.9668	.9918	.9564
	(d)(3)	.6694	.9452	.9944	.9519	.6683	.9485	.9945	.9535
	(e)	.6814	.9355	.9923	.9490	.6796	.9341	.9942	.9498
24	(a)(1)	.6306	.8530	.9906	.9167	.6126	.8525	.9741	.9061
	(a)(2)	.6262	.8594	.9889	.9178	.6051	.8607	.9744	.9085
	(b)(1)	.6147	.8958	.9910	.9308	.5968	.8720	.9763	.9127
	(b)(2)	.6080	.9007	.9905	.9315	.5891	.8770	.9760	.9134
	(c)	.9239	.5293	.9332	.7467	.9183	.5316	.9237	.7439
	(d)(1)	.6043	.9500	.9875	.9459	.5813	.9489	.9739	.9355
	(d)(2)	.5994	.9551	.9864	.9464	.5755	.9523	.9733	.9356
	(d)(3)	.6089	.9379	.9859	.9416	.5854	.9300	.9714	.9284
	(e)	.6162	.9306	.9849	.9395	.5940	.9151	.9705	.9241

Table 12

Efficiencies of Debiased Adaptive Robust Estimates of Canonical Scale Parameter

(Relative to Debiased Maximum Likelihood Estimate if Population is Known)

		Phase I				Phase II			
Sample Size		Samples from				Samples from			
n	Criterion	U	N	D	Avg.	U	N	D	Avg.
8	(a)(1)	.5674	.7960	.9398	.8494	.5866	.7887	.9420	.8526
	(a)(2)	.5685	.8393	.9570	.8725	.5906	.8357	.9604	.8783
	(b)(1)	.6375	.7957	.9311	.8564	.6726	.8042	.9401	.8699
	(b)(2)	.6257	.8343	.9495	.8770	.6500	.8380	.9629	.8902
	(c)	.9992	.9401	.9388	.9439	.9998	.9414	.9559	.9550
	(d)(1)	.7764	.8982	.9647	.9263	.7924	.8956	.9770	.9350
	(d)(2)	.7155	.9071	.9749	.9272	.7315	.9032	.9912	.9381
	(d)(3)	.7209	.8840	.9556	.9098	.7471	.8887	.9649	.9205
	(e)	.6285	.8221	.9388	.8678	.6631	.8375	.9511	.8855
12	(a)(1)	.5996	.7441	.9225	.8409	.5919	.7387	.9023	.8227
	(a)(2)	.5765	.7702	.9317	.8523	.5721	.7701	.9113	.8363
	(b)(1)	.6973	.7470	.9213	.8506	.6823	.7447	.8944	.8297
	(b)(2)	.6650	.7664	.9322	.8608	.6539	.7693	.9044	.8415
	(c)	.9973	.8679	.9205	.9091	.9948	.8708	.8673	.8751
	(d)(1)	.7916	.8432	.9336	.8977	.7585	.8385	.9003	.8716
	(d)(2)	.7317	.8514	.9451	.9026	.7158	.8513	.9111	.8785
	(d)(3)	.7317	.8255	.9325	.8868	.7116	.8222	.8934	.8583
	(e)	.6481	.7649	.9269	.8557	.6204	.7610	.8902	.8272
16	(a)(1)	.6030	.7226	.9040	.8255	.5868	.7311	.9253	.8391
	(a)(2)	.5688	.7376	.9069	.8296	.5595	.7448	.9284	.8435
	(b)(1)	.7541	.7307	.8989	.8361	.7322	.7322	.9331	.8544
	(b)(2)	.7105	.7400	.9029	.8393	.6955	.7482	.9349	.8593
	(c)	.9941	.8268	.8582	.8542	.9944	.8485	.8944	.8843
	(d)(1)	.7905	.8400	.8845	.8666	.7819	.8326	.9146	.8827
	(d)(2)	.7488	.8482	.8871	.8684	.7431	.8455	.9167	.8859
	(d)(3)	.7570	.8099	.8857	.8560	.7397	.8059	.9169	.8726
	(e)	.6898	.7585	.8913	.8379	.6720	.7557	.9175	.8507
20	(a)(1)	.6162	.7181	.8947	.8218	.6130	.7124	.9112	.8277
	(a)(2)	.5836	.7247	.8974	.8236	.5780	.7192	.9139	.8295
	(b)(1)	.7741	.7161	.8913	.8276	.7579	.7228	.9048	.8361
	(b)(2)	.7309	.7196	.8957	.8295	.7284	.7270	.9077	.8380
	(c)	.9949	.8305	.8578	.8541	.9930	.8239	.8740	.8621
	(d)(1)	.8279	.8430	.8761	.8643	.7975	.8350	.8843	.8656
	(d)(2)	.7821	.8427	.8771	.8630	.7729	.8375	.8845	.8655
	(d)(3)	.8044	.7941	.8817	.8513	.7759	.7906	.8927	.8550
	(e)	.7724	.7639	.8856	.8419	.7084	.7545	.8975	.8417
24	(a)(1)	.6160	.7148	.9010	.8246	.5827	.7017	.8863	.8093
	(a)(2)	.5899	.7194	.9013	.8251	.5669	.7046	.8862	.8095
	(b)(1)	.7867	.7229	.9038	.8372	.7599	.6908	.8902	.8153
	(b)(2)	.7643	.7248	.9045	.8375	.7457	.6947	.8904	.8165
	(c)	.9870	.8218	.8686	.8572	.9850	.8093	.8595	.8471
	(d)(1)	.8178	.8287	.8773	.8602	.8082	.8278	.8706	.8553
	(d)(2)	.8008	.8354	.8766	.8613	.7879	.8378	.8706	.8578
	(d)(3)	.7975	.7861	.8874	.8515	.7940	.7712	.8769	.8397
	(e)	.7700	.7656	.8911	.8454	.7389	.7387	.8804	.8278

TABLE 13

DEBIASING FACTORS FOR MAXIMUM LIKELIHOOD ESTIMATORS OF SCALE PARAMETER FOR DOUBLE SPIKE, ARC SINE, SYMMETRIC BETA AND STUDENT T POPULATIONS
PHASE III: N=8(4)24

POPULATION	DEBIASING FACTORS N=8	N=12	N=16	N=20	N=24
DS	1.0079	1.0005	1.0000	1.0000	1.0000
AS	1.1146	1.0537	1.0327	1.0208	1.0148
SB(1.5)	1.3183	1.2255	1.1892	1.1554	1.1355
SB(2.0)	1.2225	1.1690	1.1362	1.1130	1.0994
SB(2.5)	1.1882	1.1328	1.1086	1.0848	1.0783
SB(3.0)	1.1616	1.1163	1.0871	1.0745	1.0659
SB(3.5)	1.1553	1.0997	1.0841	1.0639	1.0546
SB(4.0)	1.1545	1.1015	1.0749	1.0597	1.0497
ST(16)	1.0801	1.0400	1.0239	1.0102	1.0055
ST(10)	1.0645	1.0233	1.0099	.9969	.9909
ST(8)	1.0490	1.0199	1.0003	.9871	.9756
ST(7)	1.0494	1.0065	.9896	.9833	.9721
ST(6)	1.0399	1.0044	.9830	.9763	.9664
ST(5)	1.0428	.9886	.9774	.9629	.9533

TABLE 14

CONTINGENCY TABLES--CLASSIFICATION VS. TRUE POPULATION BY CRITERIA
(PHASE III: N= 8)

		CRITERION (A)(1)				CRITERION (A)(2)			
		CLASSIFIED AS				CLASSIFIED AS			
		U	N	D	SUMS	U	N	D	SUMS
NUMBER OF	DS	3579	0	1421	5000	3556	1140	304	5000
SAMPLES FROM	AS	3520	686	794	5000	3202	1195	603	5000
	SB(1.5)	2720	995	1285	5000	2274	1757	969	5000
	SB(2.0)	2516	1098	1386	5000	2084	1839	1077	5000
	SB(2.5)	2416	1127	1457	5000	1980	1901	1119	5000
	SB(3.0)	2349	1066	1585	5000	1939	1842	1219	5000
	SB(3.5)	2251	1118	1631	5000	1839	1861	1300	5000
	SB(4.0)	2169	1096	1735	5000	1756	1877	1367	5000
	ST(16)	1755	1036	2209	5000	1428	1739	1833	5000
	ST(10)	1675	1037	2288	5000	1320	1810	1870	5000
	ST(8)	1590	1069	2341	5000	1243	1774	1983	5000
	ST(7)	1562	986	2452	5000	1239	1723	2038	5000
	ST(6)	1515	948	2537	5000	1172	1672	2156	5000
	ST(5)	1379	1018	2603	5000	1085	1707	2208	5000
	SUMS	30996	13280	25724	70000	26117	23837	20046	70000

		CRITERION (B)(1)				CRITERION (B)(2)			
NUMBER OF	DS	3559	1135	306	5000	3556	1140	304	5000
SAMPLES FROM	AS	3823	588	589	5000	3497	1048	455	5000
	SB(1.5)	2694	1027	1279	5000	2232	1834	934	5000
	SB(2.0)	2532	1039	1429	5000	2013	1900	1087	5000
	SB(2.5)	2403	1096	1501	5000	1909	1927	1164	5000
	SB(3.0)	2363	1063	1574	5000	1895	1862	1243	5000
	SB(3.5)	2272	1094	1634	5000	1773	1887	1340	5000
	SB(4.0)	2131	1108	1761	5000	1690	1907	1403	5000
	ST(16)	1771	1063	2166	5000	1386	1818	1796	5000
	ST(10)	1689	1058	2253	5000	1324	1791	1885	5000
	ST(8)	1617	1016	2367	5000	1232	1804	1964	5000
	ST(7)	1585	995	2420	5000	1217	1732	2051	5000
	ST(6)	1532	998	2470	5000	1160	1730	2110	5000
	ST(5)	1406	1012	2582	5000	1040	1778	2182	5000
	SUMS	31377	14292	24331	70000	25924	24158	19918	70000

TABLE 14

CONTINGENCY TABLES--CLASSIFICATION VS. TRUE POPULATION BY CRITERIA
(PHASE III: N= 8)

		CRITERION (D)(1)				CRITERION (D)(2)			
		CLASSIFIED AS				CLASSIFIED AS			
		U	N	D	SUMS	U	N	D	SUMS
NUMBER OF	DS	4690	12	298	5000	4689	13	298	5000
SAMPLES FROM	AS	4013	707	280	5000	3811	944	245	5000
	SB(1.5)	2559	1929	512	5000	2219	2313	468	5000
	SB(2.0)	2332	2079	589	5000	2000	2464	536	5000
	SB(2.5)	2253	2094	653	5000	1933	2472	595	5000
	SB(3.0)	2198	2079	723	5000	1860	2479	661	5000
	SB(3.5)	2161	2059	780	5000	1857	2431	712	5000
	SB(4.0)	1991	2183	826	5000	1709	2528	763	5000
	ST(16)	1699	2158	1143	5000	1475	2454	1071	5000
	ST(10)	1625	2189	1186	5000	1445	2456	1099	5000
	ST(8)	1585	2148	1267	5000	1390	2418	1192	5000
	ST(7)	1516	2108	1376	5000	1383	2339	1278	5000
	ST(6)	1505	2072	1423	5000	1308	2356	1336	5000
	ST(5)	1454	2074	1472	5000	1299	2330	1371	5000
	SUMS	31581	25891	12528	70000	28378	29997	11625	70000

		CRITERION (D)(3)				CRITERION (E)			
NUMBER OF	DS	3560	11	1429	5000	3560	5	1435	5000
SAMPLES FROM	AS	3884	576	540	5000	3859	425	716	5000
	SB(1.5)	2602	1680	718	5000	2564	1228	1208	5000
	SB(2.0)	2318	1870	812	5000	2288	1380	1332	5000
	SB(2.5)	2236	1873	891	5000	2203	1391	1406	5000
	SB(3.0)	2176	1863	961	5000	2142	1393	1465	5000
	SB(3.5)	2119	1840	1041	5000	2085	1367	1548	5000
	SB(4.0)	1969	1975	1056	5000	1943	1472	1585	5000
	ST(16)	1607	1984	1409	5000	1575	1359	2066	5000
	ST(10)	1578	1996	1426	5000	1548	1367	2085	5000
	ST(8)	1481	1978	1541	5000	1445	1368	2187	5000
	ST(7)	1423	1942	1635	5000	1399	1325	2276	5000
	ST(6)	1417	1912	1671	5000	1386	1311	2303	5000
	ST(5)	1327	1893	1780	5000	1283	1256	2461	5000
	SUMS	29697	23393	16910	70000	29280	16647	24073	70000

TABLE 15

CONTINGENCY TABLES--CLASSIFICATION VS. TRUE POPULATION BY CRITERIA (PHASE III: N=12)

		CRITERION (A)(1)				CRITERION (A)(2)			
		CLASSIFIED AS				CLASSIFIED AS			
		U	N	D	SUMS	U	N	D	SUMS
NUMBER OF	DS	4279	530	191	5000	4276	531	193	5000
SAMPLES FROM	AS	4110	608	282	5000	3962	800	238	5000
	SB(1.5)	2835	1493	672	5000	2528	1911	561	5000
	SB(2.0)	2627	1518	855	5000	2354	1912	734	5000
	SB(2.5)	2351	1683	966	5000	2026	2150	824	5000
	SB(3.0)	2191	1638	1171	5000	1892	2078	1030	5000
	SB(3.5)	2009	1798	1193	5000	1749	2243	1008	5000
	SB(4.0)	2006	1691	1303	5000	1711	2157	1132	5000
	ST(16)	1346	1645	2009	5000	1113	2084	1803	5000
	ST(10)	1303	1565	2132	5000	1098	1960	1942	5000
	ST(8)	1177	1506	2317	5000	972	1905	2123	5000
	ST(7)	1120	1448	2432	5000	924	1846	2230	5000
	ST(6)	1069	1405	2526	5000	884	1787	2329	5000
	ST(5)	1009	1372	2619	5000	842	1725	2433	5000
	SUMS	29432	19900	20668	70000	26331	25089	18580	70000

		CRITERION (B)(1)				CRITERION (B)(2)			
NUMBER OF	DS	4277	530	193	5000	4277	530	193	5000
SAMPLES FROM	AS	4420	455	125	5000	4304	582	114	5000
	SB(1.5)	2786	1560	654	5000	2525	1913	562	5000
	SB(2.0)	2558	1557	885	5000	2281	1951	768	5000
	SB(2.5)	2251	1726	1023	5000	1988	2120	892	5000
	SB(3.0)	2111	1635	1254	5000	1855	2028	1117	5000
	SB(3.5)	1956	1805	1239	5000	1671	2248	1081	5000
	SB(4.0)	1913	1749	1338	5000	1673	2124	1203	5000
	ST(16)	1301	1644	2055	5000	1128	1970	1902	5000
	ST(10)	1278	1582	2140	5000	1085	1921	1994	5000
	ST(8)	1154	1538	2308	5000	988	1865	2147	5000
	ST(7)	1088	1507	2405	5000	917	1830	2253	5000
	ST(6)	1046	1438	2516	5000	895	1750	2355	5000
	ST(5)	992	1384	2624	5000	854	1673	2473	5000
	SUMS	29131	20110	20759	70000	26441	24505	19054	70000

TABLE 15

CONTINGENCY TABLES--CLASSIFICATION VS. TRUE POPULATION BY CRITERIA
(PHASE III: N=12)

		CRITERION (D)(1)				CRITERION (D)(2)			
		CLASSIFIED AS				CLASSIFIED AS			
		U	N	D	SUMS	U	N	D	SUMS
NUMBER OF	DS	4807	1	192	5000	4807	1	192	5000
SAMPLES FROM	AS	4556	350	94	5000	4485	437	78	5000
	SB(1.5)	2722	1953	325	5000	2497	2231	272	5000
	SB(2.0)	2400	2149	451	5000	2194	2409	397	5000
	SB(2.5)	2126	2332	542	5000	1923	2610	467	5000
	SB(3.0)	1995	2374	631	5000	1817	2621	562	5000
	SB(3.5)	1789	2622	589	5000	1605	2869	526	5000
	SB(4.0)	1784	2485	731	5000	1604	2731	665	5000
	ST(16)	1223	2586	1191	5000	1162	2732	1106	5000
	ST(10)	1210	2465	1325	5000	1118	2645	1237	5000
	ST(8)	1125	2445	1430	5000	1022	2621	1357	5000
	ST(7)	1073	2384	1543	5000	987	2538	1475	5000
	ST(6)	1015	2340	1645	5000	973	2489	1538	5000
	ST(5)	999	2249	1752	5000	958	2385	1657	5000
	SUMS	28824	28735	12441	70000	27152	31319	11529	70000

		CRITERION (D)(3)				CRITERION (E)			
NUMBER OF	DS	4807	0	193	5000	4807	0	193	5000
SAMPLES FROM	AS	4499	320	181	5000	4494	263	243	5000
	SB(1.5)	2728	1824	448	5000	2714	1552	734	5000
	SB(2.0)	2410	2009	581	5000	2396	1664	940	5000
	SB(2.5)	2131	2188	681	5000	2114	1821	1065	5000
	SB(3.0)	2016	2224	760	5000	2001	1807	1192	5000
	SB(3.5)	1785	2477	738	5000	1768	2022	1210	5000
	SB(4.0)	1805	2319	876	5000	1789	1910	1301	5000
	ST(16)	1213	2420	1367	5000	1200	1908	1892	5000
	ST(10)	1186	2297	1517	5000	1176	1798	2026	5000
	ST(8)	1105	2270	1625	5000	1097	1721	2182	5000
	ST(7)	1020	2221	1759	5000	1012	1691	2297	5000
	ST(6)	992	2162	1846	5000	985	1640	2375	5000
	ST(5)	957	2043	2000	5000	944	1515	2541	5000
	SUMS	28654	26774	14572	70000	28497	21312	20191	70000

TABLE 16

CONTINGENCY TABLES--CLASSIFICATION VS. TRUE POPULATION BY CRITERIA
(PHASE III: N=16)

		Criterion (A)(1) Classified as U	N	D	SUMS	Criterion (A)(2) Classified as U	N	D	SUMS
NUMBER OF SAMPLES FROM	DS	4603	283	114	5000	4603	283	114	5000
	AS	4499	406	95	5000	4419	495	86	5000
	SB(1.5)	3106	1559	335	5000	2903	1797	300	5000
	SB(2.0)	2670	1819	511	5000	2469	2061	470	5000
	SB(2.5)	2328	2022	650	5000	2126	2266	608	5000
	SB(3.0)	2125	2103	772	5000	1907	2376	717	5000
	SB(3.5)	1951	2160	889	5000	1730	2436	834	5000
	SB(4.0)	1886	2140	974	5000	1697	2392	911	5000
	ST(16)	1077	1908	2015	5000	946	2117	1937	5000
	ST(10)	935	1920	2145	5000	826	2104	2070	5000
	ST(8)	855	1812	2333	5000	746	2015	2239	5000
	ST(7)	879	1693	2428	5000	769	1893	2338	5000
	ST(6)	692	1655	2653	5000	608	1824	2568	5000
	ST(5)	684	1603	2713	5000	582	1781	2637	5000
	SUMS	28290	23083	18627	70000	26331	25840	17829	70000

		Criterion (B)(1) U	N	D	SUMS	Criterion (B)(2) U	N	D	SUMS
NUMBER OF SAMPLES FROM	DS	4886	0	114	5000	4886	0	114	5000
	AS	4776	209	15	5000	4738	248	14	5000
	SB(1.5)	3042	1633	325	5000	2863	1839	298	5000
	SB(2.0)	2514	1954	532	5000	2341	2155	504	5000
	SB(2.5)	2158	2133	709	5000	1984	2342	674	5000
	SB(3.0)	1943	2237	820	5000	1760	2455	785	5000
	SB(3.5)	1800	2268	932	5000	1649	2468	883	5000
	SB(4.0)	1739	2278	983	5000	1585	2481	934	5000
	ST(16)	999	2042	1959	5000	902	2187	1911	5000
	ST(10)	901	1959	2140	5000	811	2111	2078	5000
	ST(8)	777	1898	2325	5000	676	2052	2272	5000
	ST(7)	809	1806	2385	5000	722	1963	2315	5000
	ST(6)	637	1763	2600	5000	556	1894	2550	5000
	ST(5)	626	1733	2641	5000	548	1862	2590	5000
	SUMS	27607	23913	18480	70000	26021	26057	17922	70000

TABLE 16

CONTINGENCY TABLES--CLASSIFICATION VS. TRUE POPULATION BY CRITERIA
(PHASE III: N=16)

		CRITERION (D)(1)				CRITERION (D)(2)			
		CLASSIFIED AS				CLASSIFIED AS			
		U	N	D	SUMS	U	N	D	SUMS
NUMBER OF	DS	4886	0	114	5000	4886	0	114	5000
SAMPLES FROM	AS	4867	111	22	5000	4838	147	15	5000
	SB(1.5)	2932	1885	183	5000	2752	2089	159	5000
	SB(2.0)	2394	2330	276	5000	2214	2539	247	5000
	SB(2.5)	2049	2571	380	5000	1891	2764	345	5000
	SB(3.0)	1829	2763	408	5000	1664	2946	390	5000
	SB(3.5)	1696	2839	465	5000	1560	3008	432	5000
	SB(4.0)	1629	2853	518	5000	1503	3011	486	5000
	ST(16)	986	2882	1132	5000	910	2990	1100	5000
	ST(10)	872	2777	1351	5000	820	2876	1304	5000
	ST(8)	766	2759	1475	5000	719	2841	1440	5000
	ST(7)	807	2660	1533	5000	764	2771	1465	5000
	ST(6)	656	2640	1704	5000	619	2723	1658	5000
	ST(5)	640	2542	1818	5000	605	2627	1768	5000
	SUMS	27009	31612	11379	70000	25745	33332	10923	70000

		CRITERION (D)(3)				CRITERION (E)			
NUMBER OF	DS	4886	0	114	5000	4886	0	114	5000
SAMPLES FROM	AS	4840	101	59	5000	4838	93	69	5000
	SB(1.5)	2967	1772	261	5000	2961	1584	455	5000
	SB(2.0)	2418	2213	369	5000	2411	1960	629	5000
	SB(2.5)	2088	2419	493	5000	2078	2167	755	5000
	SB(3.0)	1858	2585	557	5000	1846	2328	826	5000
	SB(3.5)	1713	2652	635	5000	1710	2351	939	5000
	SB(4.0)	1643	2658	699	5000	1635	2378	987	5000
	ST(16)	970	2567	1463	5000	966	2194	1840	5000
	ST(10)	859	2480	1661	5000	852	2122	2026	5000
	ST(8)	738	2437	1825	5000	730	2054	2216	5000
	ST(7)	788	2348	1864	5000	784	1968	2248	5000
	ST(6)	626	2310	2064	5000	621	1939	2440	5000
	ST(5)	609	2203	2188	5000	601	1812	2587	5000
	SUMS	27003	28745	14252	70000	26919	24950	18131	70000

TABLE 17

CONTINGENCY TABLES--CLASSIFICATION VS. TRUE POPULATION BY CRITERIA
(PHASE III: N=20)

		CRITERION (A)(1)				CRITERION (A)(2)			
		CLASSIFIED AS				CLASSIFIED AS			
		U	N	D	SUMS	U	N	D	SUMS
NUMBER OF	DS	4784	146	70	5000	4784	146	70	5000
SAMPLES FROM	AS	4660	303	37	5000	4619	344	37	5000
	SB(1.5)	3187	1635	178	5000	3020	1814	166	5000
	SB(2.0)	2695	2004	301	5000	2495	2217	288	5000
	SB(2.5)	2257	2294	449	5000	2077	2491	432	5000
	SB(3.0)	2025	2431	544	5000	1867	2612	521	5000
	SB(3.5)	1881	2493	626	5000	1719	2672	609	5000
	SB(4.0)	1731	2512	757	5000	1562	2700	738	5000
	ST(16)	851	2184	1965	5000	767	2306	1927	5000
	ST(10)	747	2046	2207	5000	650	2177	2173	5000
	ST(8)	620	1987	2393	5000	555	2079	2366	5000
	ST(7)	601	1881	2518	5000	542	1966	2492	5000
	ST(6)	506	1831	2663	5000	453	1915	2632	5000
	ST(5)	473	1678	2849	5000	407	1774	2819	5000
	SUMS	27018	25425	17557	70000	25517	27213	17270	70000

		CRITERION (B)(1)				CRITERION (B)(2)			
NUMBER OF	DS	4930	56	14	5000	4930	56	14	5000
SAMPLES FROM	AS	4896	102	2	5000	4882	116	2	5000
	SB(1.5)	3107	1730	163	5000	2972	1869	159	5000
	SB(2.0)	2504	2179	317	5000	2353	2339	308	5000
	SB(2.5)	2076	2447	477	5000	1949	2589	462	5000
	SB(3.0)	1835	2593	572	5000	1711	2727	562	5000
	SB(3.5)	1693	2662	645	5000	1581	2783	636	5000
	SB(4.0)	1551	2640	809	5000	1446	2767	787	5000
	ST(16)	760	2327	1913	5000	686	2422	1892	5000
	ST(10)	672	2175	2153	5000	607	2265	2128	5000
	ST(8)	566	2058	2376	5000	507	2142	2351	5000
	ST(7)	524	1993	2483	5000	477	2066	2457	5000
	ST(6)	496	1930	2574	5000	452	2001	2547	5000
	ST(5)	417	1798	2785	5000	378	1856	2766	5000
	SUMS	26027	26690	17283	70000	24931	27998	17071	70000

TABLE 17

CONTINGENCY TABLES--CLASSIFICATION VS. TRUE POPULATION BY CRITERIA
(PHASE III: N=20)

		CRITERION (D)(1)				CRITERION (D)(2)			
		CLASSIFIED AS				CLASSIFIED AS			
		U	N	D	SUMS	U	N	D	SUMS
NUMBER OF	DS	4930	0	70	5000	4986	0	14	5000
SAMPLES FROM	AS	4937	55	8	5000	4929	68	3	5000
	SB(1.5)	3017	1845	138	5000	2885	1992	123	5000
	SB(2.0)	2379	2434	187	5000	2239	2588	173	5000
	SB(2.5)	1986	2738	276	5000	1843	2906	251	5000
	SB(3.0)	1742	2946	312	5000	1609	3103	288	5000
	SB(3.5)	1621	3028	351	5000	1499	3173	328	5000
	SB(4.0)	1463	3122	415	5000	1368	3249	383	5000
	ST(16)	728	3130	1142	5000	680	3205	1115	5000
	ST(10)	640	3055	1305	5000	588	3131	1281	5000
	ST(8)	533	2892	1575	5000	512	2952	1536	5000
	ST(7)	531	2825	1644	5000	516	2876	1608	5000
	ST(6)	502	2717	1781	5000	462	2784	1754	5000
	ST(5)	433	2580	1987	5000	420	2624	1956	5000
	SUMS	25442	33367	11191	70000	24536	34651	10813	70000

		CRITERION (D)(3)				CRITERION (E)			
NUMBER OF	DS	4930	0	70	5000	4930	0	70	5000
SAMPLES FROM	AS	4919	55	26	5000	4918	50	32	5000
	SB(1.5)	3047	1769	184	5000	3040	1669	291	5000
	SB(2.0)	2393	2335	272	5000	2387	2199	414	5000
	SB(2.5)	2019	2594	387	5000	2012	2421	567	5000
	SB(3.0)	1757	2786	457	5000	1746	2590	664	5000
	SB(3.5)	1637	2852	511	5000	1631	2677	692	5000
	SB(4.0)	1480	2919	601	5000	1476	2745	779	5000
	ST(16)	716	2752	1532	5000	713	2499	1788	5000
	ST(10)	647	2627	1726	5000	642	2403	1955	5000
	ST(8)	535	2499	1966	5000	532	2276	2192	5000
	ST(7)	527	2420	2053	5000	520	2181	2299	5000
	ST(6)	493	2280	2227	5000	487	2060	2453	5000
	ST(5)	426	2178	2396	5000	420	1964	2616	5000
	SUMS	25526	30066	14408	70000	25454	27734	16812	70000

TABLE 18

CONTINGENCY TABLES--CLASSIFICATION VS. TRUE POPULATION BY CRITERIA
(PHASE III: N=24)

		CRITERION (A)(1)				CRITERION (A)(2)			
		CLASSIFIED AS				CLASSIFIED AS			
		U	N	D	SUMS	U	N	D	SUMS
NUMBER OF	DS	4895	75	30	5000	4895	75	30	5000
SAMPLES FROM	AS	4774	214	12	5000	4755	233	12	5000
	SB(1.5)	3303	1624	73	5000	3225	1705	70	5000
	SB(2.0)	2595	2246	159	5000	2507	2338	155	5000
	SB(2.5)	2253	2459	288	5000	2172	2544	284	5000
	SB(3.0)	1933	2712	355	5000	1850	2801	349	5000
	SB(3.5)	1766	2807	427	5000	1678	2904	418	5000
	SB(4.0)	1567	2912	521	5000	1486	3003	511	5000
	ST(16)	661	2516	1823	5000	625	2564	1811	5000
	ST(10)	527	2339	2134	5000	503	2375	2122	5000
	ST(8)	457	2226	2317	5000	430	2261	2309	5000
	ST(7)	427	2091	2482	5000	401	2129	2470	5000
	ST(6)	379	1903	2718	5000	354	1939	2707	5000
	ST(5)	305	1780	2915	5000	288	1814	2898	5000
	SUMS	25842	27904	16254	70000	25169	28685	16146	70000

		CRITERION (B)(1)				CRITERION (B)(2)			
NUMBER OF	DS	4970	24	6	5000	4970	24	6	5000
SAMPLES FROM	AS	4943	57	0	5000	4938	62	0	5000
	SB(1.5)	3189	1737	74	5000	3108	1818	74	5000
	SB(2.0)	2427	2396	177	5000	2355	2470	175	5000
	SB(2.5)	2038	2666	296	5000	1963	2744	293	5000
	SB(3.0)	1752	2871	377	5000	1685	2942	373	5000
	SB(3.5)	1576	2954	470	5000	1501	3034	465	5000
	SB(4.0)	1405	3053	542	5000	1338	3123	539	5000
	ST(16)	597	2619	1784	5000	565	2662	1773	5000
	ST(10)	468	2438	2094	5000	451	2464	2085	5000
	ST(8)	411	2305	2284	5000	396	2332	2272	5000
	ST(7)	378	2195	2427	5000	360	2221	2419	5000
	ST(6)	342	2030	2628	5000	326	2052	2622	5000
	ST(5)	274	1903	2823	5000	254	1927	2819	5000
	SUMS	24770	29248	15982	70000	24210	29875	15915	70000

TABLE 18

CONTINGENCY TABLES--CLASSIFICATION VS. TRUE POPULATION BY CRITERIA
(PHASE III: N=24)

		CRITERION (D)(1)				CRITERION (D)(2)			
		CLASSIFIED AS				CLASSIFIED AS			
		U	N	D	SUMS	U	N	D	SUMS
NUMBER OF	DS	4994	0	6	5000	4994	0	6	5000
SAMPLES FROM	AS	4979	19	2	5000	4975	24	1	5000
	SB(1.5)	3087	1830	83	5000	2962	1970	68	5000
	SB(2.0)	2280	2590	130	5000	2168	2712	120	5000
	SB(2.5)	1881	2924	195	5000	1793	3029	178	5000
	SB(3.0)	1626	3132	242	5000	1545	3230	225	5000
	SB(3.5)	1409	3295	296	5000	1314	3413	273	5000
	SB(4.0)	1265	3391	344	5000	1196	3484	320	5000
	ST(16)	541	3308	1151	5000	507	3379	1114	5000
	ST(10)	467	3097	1436	5000	434	3178	1388	5000
	ST(8)	402	2968	1630	5000	383	3032	1585	5000
	ST(7)	354	2935	1711	5000	344	2991	1665	5000
	ST(6)	318	2731	1951	5000	305	2795	1900	5000
	ST(5)	268	2526	2206	5000	261	2585	2154	5000
	SUMS	23871	34746	11383	70000	23181	35822	10997	70000

		CRITERION (D)(3)				CRITERION (E)			
NUMBER OF	DS	4970	0	30	5000	4970	0	30	5000
SAMPLES FROM	AS	4975	18	7	5000	4975	15	10	5000
	SB(1.5)	3112	1778	110	5000	3108	1720	172	5000
	SB(2.0)	2306	2506	188	5000	2303	2416	281	5000
	SB(2.5)	1892	2827	281	5000	1890	2726	384	5000
	SB(3.0)	1636	3018	346	5000	1636	2906	458	5000
	SB(3.5)	1421	3164	415	5000	1419	3052	529	5000
	SB(4.0)	1274	3222	504	5000	1274	3091	635	5000
	ST(16)	542	2914	1544	5000	541	2775	1684	5000
	ST(10)	473	2708	1819	5000	471	2568	1961	5000
	ST(8)	409	2563	2028	5000	409	2431	2160	5000
	ST(7)	343	2501	2156	5000	343	2381	2276	5000
	ST(6)	315	2311	2374	5000	314	2192	2494	5000
	ST(5)	265	2109	2626	5000	263	1993	2744	5000
	SUMS	23933	31639	14428	70000	23916	30266	15818	70000

TABLE 19

MEAN SQUARE ERRORS OF PARAMETER ESTIMATES (PHASE III) IF POPULATION IS KNOWN

SAMPLE SIZE, N	POPULATION	MSE($\hat{\mu}$)	MSE($\hat{\sigma}$)	MSE($F\hat{\sigma}$)	MSE($\bar{\sigma}$)	MSE($F\bar{\sigma}$)
8	DS	.0056	.0056	.0056	.0057	.0057
	AS	.0285	.0219	.0435	.0148	.0294
	SB(1.5)	.0929	.0623	.1923	.0466	.1438
	SB(2.0)	.1032	.0564	.1857	.0442	.1455
	SB(2.5)	.1084	.0571	.1950	.0481	.1643
	SB(3.0)	.1163	.0592	.2070	.0523	.1829
	SB(3.5)	.1210	.0605	.2149	.0561	.1993
	SB(4.0)	.1214	.0605	.2174	.0574	.2063
	ST(16)	.1274	.0790	.3107	.0847	.3331
	ST(10)	.1426	.0870	.3455	.0945	.3753
	ST(8)	.1431	.0994	.3964	.1064	.4244
	ST(7)	.1467	.0996	.3978	.1073	.4286
	ST(6)	.1475	.1164	.4646	.1240	.4950
	ST(5)	.1579	.1410	.5590	.1517	.6015
	AVG	.1116	.0718	.2668	.0710	.2668
12	DS	.0006	.0006	.0006	.0006	.0006
	AS	.0092	.0067	.0133	.0044	.0087
	SB(1.5)	.0530	.0337	.1040	.0246	.0759
	SB(2.0)	.0629	.0337	.1109	.0271	.0892
	SB(2.5)	.0698	.0334	.1141	.0284	.0970
	SB(3.0)	.0762	.0345	.1206	.0304	.1063
	SB(3.5)	.0751	.0349	.1240	.0318	.1130
	SB(4.0)	.0772	.0367	.1319	.0347	.1247
	ST(16)	.0875	.0541	.2127	.0569	.2237
	ST(10)	.0954	.0597	.2371	.0620	.2462
	ST(8)	.0930	.0689	.2748	.0712	.2840
	ST(7)	.0974	.0735	.2936	.0744	.2972
	ST(6)	.0951	.0876	.3497	.0884	.3529
	ST(5)	.1015	.1044	.4139	.1021	.4048
	AVG	.0710	.0473	.1787	.0455	.1732

TABLE 19

MEAN SQUARE ERRORS OF PARAMETER ESTIMATES (PHASE III) IF POPULATION IS KNOWN

SAMPLE SIZE, N	POPULATION	MSE($\hat{\mu}$)	MSE($\hat{\sigma}$)	MSE($F\hat{\sigma}$)	MSE($\bar{\sigma}$)	MSE($F\bar{\sigma}$)
16	DS	0.0000	0.0000	0.0000	0.0000	0.0000
	AS	.0035	.0025	.0050	.0017	.0034
	SB(1.5)	.0351	.0222	.0685	.0174	.0537
	SB(2.0)	.0459	.0219	.0721	.0184	.0606
	SB(2.5)	.0518	.0230	.0786	.0202	.0690
	SB(3.0)	.0539	.0232	.0811	.0209	.0731
	SB(3.5)	.0576	.0247	.0877	.0225	.0799
	SB(4.0)	.0574	.0253	.0909	.0236	.0848
	ST(16)	.0669	.0383	.1506	.0396	.1557
	ST(10)	.0685	.0461	.1831	.0471	.1871
	ST(8)	.0681	.0511	.2038	.0511	.2038
	ST(7)	.0727	.0559	.2233	.0548	.2189
	ST(6)	.0728	.0666	.2658	.0641	.2559
	ST(5)	.0747	.0819	.3247	.0777	.3081
	AVG	.0521	.0345	.1311	.0328	.1253
20	DS	0.0000	0.0000	0.0000	0.0000	0.0000
	AS	.0017	.0012	.0024	.0008	.0016
	SB(1.5)	.0270	.0162	.0500	.0120	.0370
	SB(2.0)	.0344	.0164	.0540	.0137	.0451
	SB(2.5)	.0401	.0171	.0584	.0142	.0485
	SB(3.0)	.0431	.0180	.0629	.0161	.0563
	SB(3.5)	.0457	.0197	.0700	.0183	.0650
	SB(4.0)	.0484	.0207	.0744	.0192	.0690
	ST(16)	.0538	.0312	.1227	.0316	.1243
	ST(10)	.0538	.0359	.1426	.0356	.1414
	ST(8)	.0579	.0426	.1699	.0412	.1643
	ST(7)	.0586	.0483	.1929	.0465	.1857
	ST(6)	.0570	.0527	.2104	.0497	.1984
	ST(5)	.0607	.0721	.2859	.0657	.2605
	AVG	.0416	.0280	.1069	.0260	.0998

TABLE 19

MEAN SQUARE ERRORS OF PARAMETER ESTIMATES (PHASE III) IF POPULATION IS KNOWN

SAMPLE SIZE,N	POPULATION	MSE($\hat{\mu}$)	MSE($\hat{\sigma}$)	MSE($F\hat{\sigma}$)	MSE($\overline{\sigma}$)	MSE($F\overline{\sigma}$)
24	DS	0.0000	0.0000	0.0000	0.0000	0.0000
	AS	.0009	.0006	.0012	.0004	.0008
	SB(1.5)	.0202	.0121	.0373	.0097	.0299
	SB(2.0)	.0279	.0129	.0425	.0106	.0349
	SB(2.5)	.0327	.0139	.0475	.0120	.0410
	SB(3.0)	.0343	.0148	.0517	.0133	.0465
	SB(3.5)	.0371	.0156	.0554	.0145	.0515
	SB(4.0)	.0376	.0157	.0564	.0148	.0532
	ST(16)	.0453	.0265	.1042	.0268	.1054
	ST(10)	.0467	.0311	.1235	.0304	.1207
	ST(8)	.0469	.0370	.1476	.0349	.1392
	ST(7)	.0462	.0400	.1598	.0374	.1494
	ST(6)	.0493	.0471	.1880	.0429	.1712
	ST(5)	.0502	.0639	.2534	.0567	.2248
	AVG	.0340	.0237	.0906	.0217	.0835

TABLE 20

EFFICIENCIES OF ADAPTIVE ROBUST ESTIMATES OF
LOCATION PARAMETER
(RELATIVE TO MAXIMUM LIKELIHOOD ESTIMATE IF POPULATION IS KNOWN)
(PHASE III)

SAMPLE SIZE,N	SAMPLES FROM	CRITERION (A)(1)	(A)(2)	(B)(1)	(B)(2)	(D)(1)	(D)(2)	(D)(3)	(E)
8	DS	.0193	.0465	.0465	.0465	.0875	.0875	.0193	.0193
	AS	.1997	.2082	.2649	.2634	.3760	.3780	.2531	.2405
	SB(1.5)	.5974	.6173	.6372	.6570	.7218	.7258	.6579	.6140
	SB(2.0)	.6611	.6866	.6898	.7192	.7690	.7812	.7088	.6723
	SB(2.5)	.7285	.7481	.7450	.7645	.7976	.8102	.7607	.7207
	SB(3.0)	.7347	.7692	.7498	.7816	.8004	.8190	.7651	.7324
	SB(3.5)	.7629	.7878	.7548	.7908	.8061	.8192	.7658	.7392
	SB(4.0)	.7564	.7863	.7503	.7909	.8104	.8259	.7732	.7421
	ST(16)	.8476	.8798	.8235	.8626	.8079	.8257	.8230	.8079
	ST(10)	.8963	.9314	.8468	.9008	.8079	.8098	.8503	.8418
	ST(8)	.8844	.9203	.8388	.8828	.7977	.7924	.8373	.8349
	ST(7)	.9072	.9428	.8594	.9078	.8205	.8056	.8670	.8579
	ST(6)	.9755	1.0041	.9253	.9666	.8213	.8185	.9230	.9225
	ST(5)	1.0194	1.0492	.9777	1.0220	.8531	.8306	.9570	.9587
	AVG	.6766	.7553	.7331	.7632	.7656	.7684	.6835	.6667
12	DS	.0092	.0092	.0092	.0092	.0155	.0155	.0155	.0155
	AS	.1444	.1435	.2493	.2353	.3297	.3370	.2341	.2249
	SB(1.5)	.5928	.5962	.6424	.6448	.6883	.6910	.6386	.6127
	SB(2.0)	.6485	.6614	.6793	.6837	.7172	.7263	.6807	.6511
	SB(2.5)	.7079	.7188	.7174	.7271	.7497	.7579	.7188	.6810
	SB(3.0)	.7463	.7612	.7545	.7651	.7807	.7938	.7628	.7264
	SB(3.5)	.7488	.7687	.7473	.7695	.7897	.8075	.7758	.7334
	SB(4.0)	.7452	.7689	.7481	.7674	.7674	.7902	.7576	.7318
	ST(16)	.8528	.8785	.8087	.8341	.7933	.7806	.8162	.8087
	ST(10)	.8665	.8883	.8281	.8548	.7937	.7950	.8231	.8175
	ST(8)	.8934	.9163	.8447	.8716	.8230	.8059	.8485	.8394
	ST(7)	.9163	.9456	.8673	.8944	.8130	.7938	.8604	.8574
	ST(6)	.9567	.9794	.9153	.9388	.8661	.8270	.8989	.8930
	ST(5)	1.0130	1.0336	.9630	.9864	.9079	.8735	.9603	.9566
	AVG	.7402	.7555	.7448	.7595	.7630	.7589	.7607	.7417

TABLE 20

EFFICIENCIES OF ADAPTIVE ROBUST ESTIMATES OF
LOCATION PARAMETER
(RELATIVE TO MAXIMUM LIKELIHOOD ESTIMATE IF POPULATION IS KNOWN)
(PHASE III)

SAMPLE SIZE,N	SAMPLES FROM	CRITERION (A)(1)	(A)(2)	(B)(1)	(B)(2)	(D)(1)	(D)(2)	(D)(3)	(E)
16	DS	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AS	.1151	.1097	.2632	.2465	.3977	.3977	.2482	.2431
	SB(1.5)	.6021	.5980	.6429	.6417	.6724	.6763	.6452	.6158
	SB(2.0)	.6691	.6701	.6923	.6944	.7138	.7194	.6871	.6623
	SB(2.5)	.7175	.7214	.7348	.7421	.7629	.7708	.7430	.7096
	SB(3.0)	.7414	.7528	.7434	.7486	.7581	.7744	.7497	.7254
	SB(3.5)	.7471	.7629	.7549	.7660	.7752	.7826	.7619	.7356
	SB(4.0)	.7455	.7593	.7533	.7664	.7810	.7917	.7553	.7340
	ST(16)	.8209	.8342	.8031	.8209	.7964	.8099	.8060	.7936
	ST(10)	.8616	.8760	.8263	.8436	.8223	.8012	.8415	.8263
	ST(8)	.9020	.9129	.8833	.8949	.8513	.8481	.8810	.8708
	ST(7)	.9238	.9429	.8920	.9042	.8463	.8309	.8748	.8655
	ST(6)	.9406	.9492	.9180	.9286	.8708	.8636	.9066	.9123
	ST(5)	.9829	1.0000	.9361	.9516	.9200	.9011	.9234	.9338
	AVG	.7578	.7657	.7784	.7873	.7813	.7810	.7774	.7620
20	DS	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AS	.0983	.0934	.2931	.2656	.4048	.4359	.2576	.2537
	SB(1.5)	.5684	.5649	.6164	.6095	.6236	.6250	.6027	.5857
	SB(2.0)	.6641	.6628	.6908	.6880	.6978	.7006	.6758	.6590
	SB(2.5)	.6926	.6938	.7123	.7161	.7110	.7199	.7023	.6843
	SB(3.0)	.7509	.7575	.7601	.7655	.7752	.7908	.7588	.7317
	SB(3.5)	.7591	.7642	.7566	.7642	.7707	.7785	.7591	.7383
	SB(4.0)	.7610	.7695	.7695	.7769	.7844	.7934	.7744	.7551
	ST(16)	.8446	.8526	.8201	.8367	.8354	.8252	.8264	.8152
	ST(10)	.8526	.8650	.8341	.8433	.8526	.8526	.8472	.8419
	ST(8)	.9033	.9176	.8826	.8935	.8921	.8746	.8935	.8908
	ST(7)	.9185	.9258	.8919	.9015	.8933	.8720	.8960	.8947
	ST(6)	.9238	.9360	.8851	.8892	.8906	.8837	.8906	.8906
	ST(5)	1.0202	1.0323	.9951	1.0100	.9712	.9514	.9790	.9806
	AVG	.7733	.7783	.7983	.8041	.7994	.8098	.7892	.7775

TABLE 20

EFFICIENCIES OF ADAPTIVE ROBUST ESTIMATES OF LOCATION PARAMETER
(RELATIVE TO MAXIMUM LIKELIHOOD ESTIMATE IF POPULATION IS KNOWN)
(PHASE III)

SAMPLE SIZE,N	SAMPLES FROM	CRITERION (A)(1)	(A)(2)	(B)(1)	(B)(2)	(D)(1)	(D)(2)	(D)(3)	(E)
24	DS	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AS	.0909	.0882	.3000	.2813	.6000	.5625	.4091	.3913
	SB(1.5)	.5674	.5627	.5959	.5924	.6012	.5994	.5976	.5805
	SB(2.0)	.6627	.6596	.6788	.6788	.6838	.6838	.6723	.6565
	SB(2.5)	.6972	.6957	.7093	.7063	.7078	.7140	.6972	.6813
	SB(3.0)	.7221	.7252	.7392	.7424	.7505	.7522	.7361	.7206
	SB(3.5)	.7495	.7541	.7450	.7495	.7618	.7729	.7510	.7361
	SB(4.0)	.7627	.7689	.7658	.7721	.7737	.7817	.7642	.7535
	ST(16)	.8547	.8547	.8312	.8389	.8404	.8436	.8389	.8358
	ST(10)	.8778	.8811	.8616	.8713	.8713	.8778	.8553	.8491
	ST(8)	.9196	.9250	.9142	.9142	.9054	.9054	.9002	.8899
	ST(7)	.9077	.9112	.8868	.8971	.9077	.9006	.8953	.8868
	ST(6)	.9463	.9517	.9302	.9337	.9499	.9444	.9373	.9373
	ST(5)	1.0183	1.0266	1.0000	1.0080	1.0101	.9901	1.0101	1.0121
	AVG	.7922	.7940	.8090	.8126	.8222	.8227	.8053	.7952

TABLE 21

EFFICIENCIES OF ADAPTIVE ROBUST ESTIMATES OF
SCALE PARAMETER
(RELATIVE TO MAXIMUM LIKELIHOOD ESTIMATE IF POPULATION IS KNOWN)
(PHASE III)

SAMPLE SIZE,N	SAMPLES FROM	CRITERION (A)(1)	(A)(2)	(B)(1)	(B)(2)	(D)(1)	(D)(2)	(D)(3)	(E)
8	DS	.0318	.0348	.0348	.0348	.0284	.0284	.0318	.0318
	AS	.3288	.3417	.3093	.3211	.2955	.3042	.3012	.3063
	SB(1.5)	1.0613	1.0930	1.0650	1.0968	1.0332	1.0723	1.0298	1.0524
	SB(2.0)	.9674	.9930	.9559	.9930	.9416	.9724	.9447	.9608
	SB(2.5)	.9136	.9376	.9078	.9391	.9021	.9269	.9049	.9107
	SB(3.0)	.8757	.9036	.8732	.8983	.8719	.8916	.8706	.8757
	SB(3.5)	.8705	.8897	.8594	.8897	.8521	.8680	.8557	.8594
	SB(4.0)	.8545	.8743	.8557	.8768	.8521	.8680	.8509	.8545
	ST(16)	.8485	.8672	.8541	.8768	.8720	.8837	.8749	.8662
	ST(10)	.8546	.8850	.8597	.8806	.9025	.9139	.8932	.8815
	ST(8)	.8971	.9247	.9036	.9298	.9413	.9558	.9342	.9170
	ST(7)	.9005	.9154	.9063	.9257	.9477	.9559	.9396	.9239
	ST(6)	.9319	.9557	.9387	.9636	.9864	1.0009	.9700	.9494
	ST(5)	1.0658	1.0821	1.0674	1.0905	1.1102	1.1253	1.0990	1.0780
	AVG	.7716	.7982	.7793	.7984	.7694	.7815	.7795	.7756
12	DS	.0036	.0036	.0036	.0036	.0032	.0032	.0032	.0032
	AS	.1348	.1370	.1274	.1288	.1245	.1255	.1255	.1264
	SB(1.5)	1.1049	1.1159	1.1013	1.1086	1.0564	1.0698	1.0631	1.0871
	SB(2.0)	1.0306	1.0337	1.0243	1.0243	.9912	1.0000	.9941	1.0120
	SB(2.5)	.9911	.9940	.9795	.9882	.9570	.9625	.9625	.9681
	SB(3.0)	.9557	.9610	.9504	.9504	.9249	.9324	.9299	.9401
	SB(3.5)	.9536	.9588	.9510	.9562	.9357	.9407	.9382	.9357
	SB(4.0)	.9315	.9362	.9221	.9291	.9129	.9152	.9129	.9152
	ST(16)	.8811	.8942	.8869	.8957	.9280	.9328	.9232	.9032
	ST(10)	.8884	.8991	.8991	.9073	.9461	.9506	.9416	.9171
	ST(8)	.9374	.9517	.9477	.9569	.9928	.9971	.9871	.9663
	ST(7)	.9671	.9787	.9761	.9839	1.0223	1.0265	1.0138	.9919
	ST(6)	1.0318	1.0429	1.0330	1.0478	1.0882	1.0923	1.0801	1.0554
	ST(5)	1.1447	1.1574	1.1498	1.1626	1.1986	1.2026	1.1904	1.1676
	AVG	.7522	.7587	.7513	.7567	.7490	.7521	.7476	.7414

TABLE 21

EFFICIENCIES OF ADAPTIVE ROBUST ESTIMATES OF
SCALE PARAMETER
(RELATIVE TO MAXIMUM LIKELIHOOD ESTIMATE IF POPULATION IS KNOWN)
(PHASE III)

SAMPLE SIZE,N	SAMPLES FROM	CRITERION (A)(1)	(A)(2)	(B)(1)	(B)(2)	(D)(1)	(D)(2)	(D)(3)	(E)
16	DS	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AS	.0590	.0595	.0564	.0566	.0559	.0559	.0561	.0562
	SB(1.5)	1.1563	1.1563	1.1503	1.1503	1.1045	1.1100	1.1156	1.1385
	SB(2.0)	1.0950	1.0896	1.0896	1.0896	1.0379	1.0429	1.0429	1.0580
	SB(2.5)	1.0314	1.0314	1.0268	1.0268	.9871	.9871	.9957	1.0088
	SB(3.0)	1.0131	1.0175	1.0131	1.0043	.9707	.9707	.9789	.9831
	SB(3.5)	.9880	.9920	.9920	.9880	.9648	.9648	.9724	.9841
	SB(4.0)	.9731	.9768	.9731	.9731	.9547	.9547	.9583	.9693
	ST(16)	.8928	.9012	.9076	.9097	.9672	.9672	.9504	.9364
	ST(10)	.9147	.9220	.9276	.9294	.9872	.9893	.9705	.9486
	ST(8)	.9534	.9569	.9551	.9623	1.0119	1.0119	.9980	.9733
	ST(7)	.9688	.9739	.9739	.9790	1.0508	1.0527	1.0352	1.0127
	ST(6)	1.0571	1.0622	1.0605	1.0639	1.1288	1.1307	1.1082	1.0882
	ST(5)	1.1904	1.1956	1.2062	1.2080	1.2581	1.2581	1.2409	1.2188
	AVG	.7042	.7067	.6957	.6967	.7102	.7107	.7061	.7013
20	DS	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AS	.0304	.0305	.0294	.0294	.0293	.0293	.0293	.0294
	SB(1.5)	1.1912	1.1825	1.1739	1.1655	1.1408	1.1329	1.1489	1.1655
	SB(2.0)	1.1007	1.1007	1.1007	1.1007	1.0581	1.0581	1.0649	1.0789
	SB(2.5)	1.0688	1.0688	1.0621	1.0621	1.0240	1.0240	1.0364	1.0427
	SB(3.0)	1.0112	1.0169	1.0227	1.0227	1.0000	.9945	1.0056	1.0169
	SB(3.5)	1.0103	1.0103	1.0103	1.0103	.9899	.9899	1.0000	1.0000
	SB(4.0)	1.0000	1.0049	1.0000	1.0049	.9810	.9810	.9952	1.0000
	ST(16)	.8940	.8966	.9096	.9096	.9750	.9750	.9455	.9286
	ST(10)	.8953	.9020	.9158	.9182	1.0000	1.0000	.9651	.9472
	ST(8)	.9363	.9383	.9425	.9467	1.0216	1.0216	.9953	.9816
	ST(7)	.9837	.9857	.9938	.9938	1.0639	1.0639	1.0343	1.0190
	ST(6)	1.0333	1.0354	1.0394	1.0436	1.1118	1.1118	1.0777	1.0646
	ST(5)	1.2118	1.2138	1.2138	1.2159	1.2921	1.2921	1.2671	1.2561
	AVG	.6571	.6583	.6552	.6559	.6716	.6715	.6645	.6613

TABLE 21

EFFICIENCIES OF ADAPTIVE ROBUST ESTIMATES OF
SCALE PARAMETER
(RELATIVE TO MAXIMUM LIKELIHOOD ESTIMATE IF POPULATION IS KNOWN)
(PHASE III)

SAMPLE SIZE,N	SAMPLES FROM	CRITERION (A)(1)	(A)(2)	(B)(1)	(B)(2)	(D)(1)	(D)(2)	(D)(3)	(E)
24	DS	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AS	.0159	.0160	.0156	.0156	.0156	.0156	.0156	.0156
	SB(1.5)	1.1524	1.1524	1.1415	1.1308	1.1204	1.1101	1.1308	1.1415
	SB(2.0)	1.0488	1.0488	1.0661	1.0661	1.0320	1.0320	1.0488	1.0488
	SB(2.5)	1.0000	1.0000	1.0146	1.0146	.9858	.9858	1.0000	1.0072
	SB(3.0)	1.0000	1.0000	1.0137	1.0137	.9867	.9867	1.0000	1.0068
	SB(3.5)	.9689	.9750	.9873	.9936	.9750	.9750	.9811	.9811
	SB(4.0)	.9632	.9691	.9752	.9813	.9691	.9691	.9813	.9813
	ST(16)	.8689	.8717	.8863	.8893	.9636	.9672	.9331	.9233
	ST(10)	.8937	.8937	.9067	.9094	.9873	.9904	.9569	.9424
	ST(8)	.9343	.9343	.9415	.9415	1.0137	1.0193	.9814	.9686
	ST(7)	.9547	.9547	.9662	.9662	1.0390	1.0417	.9975	.9901
	ST(6)	1.0195	1.0217	1.0374	1.0397	1.1108	1.1135	1.0753	1.0729
	ST(5)	1.2704	1.2704	1.2883	1.2883	1.3424	1.3453	1.3148	1.3067
	AVG	.6119	.6124	.6140	.6145	.6298	.6305	.6226	.6206

TABLE 22

EFFICIENCIES OF ADAPTIVE ROBUST ESTIMATES OF
CANONICAL SCALE PARAMETER
(RELATIVE TO MAXIMUM LIKELIHOOD ESTIMATE IF POPULATION IS KNOWN)
(PHASE III)

SAMPLE SIZE,N	SAMPLES FROM	CRITERION (A)(1)	(A)(2)	(B)(1)	(B)(2)	(D)(1)	(D)(2)	(D)(3)	(E)
8	DS	.0836	.0463	.0463	.0463	.4828	.4828	.0836	.0836
	AS	.4651	.4721	.6122	.6114	.7416	.7328	.6439	.5979
	SB(1.5)	.8613	.9485	.8831	.9784	.9447	1.0013	.9238	.8896
	SB(2.0)	.7468	.8139	.7365	.8248	.7941	.8412	.7887	.7659
	SB(2.5)	.6887	.7542	.6858	.7630	.7362	.7786	.7346	.7094
	SB(3.0)	.6583	.7222	.6594	.7232	.7043	.7419	.6983	.6811
	SB(3.5)	.6625	.7155	.6547	.7212	.6841	.7138	.6856	.6691
	SB(4.0)	.6417	.6960	.6496	.7052	.6878	.7192	.6822	.6657
	ST(16)	.6509	.6899	.6561	.7044	.6919	.7183	.6970	.6802
	ST(10)	.6508	.7040	.6549	.6986	.7182	.7442	.7042	.6849
	ST(8)	.6980	.7469	.7022	.7541	.7582	.7893	.7504	.7258
	ST(7)	.6996	.7342	.7043	.7459	.7666	.7873	.7558	.7322
	ST(6)	.7277	.7712	.7337	.7815	.8041	.8333	.7823	.7519
	ST(5)	.8446	.8815	.8456	.8936	.9114	.9437	.8963	.8642
	AVG	.6997	.7393	.6985	.7506	.7686	.7997	.7500	.7267
12	DS	.0108	.0108	.0108	.0108	.1333	.1333	.1333	.1333
	AS	.3141	.2999	.4773	.4592	.5202	.5182	.4756	.4514
	SB(1.5)	.8919	.9361	.9146	.9480	.9335	.9558	.9319	.8965
	SB(2.0)	.7521	.7868	.7506	.7840	.7862	.8068	.7774	.7486
	SB(2.5)	.7152	.7525	.7064	.7442	.7360	.7525	.7365	.7012
	SB(3.0)	.6861	.7197	.6769	.7050	.7087	.7310	.7070	.6850
	SB(3.5)	.6677	.7081	.6680	.7077	.7154	.7384	.7109	.6727
	SB(4.0)	.6671	.7038	.6604	.6945	.7000	.7172	.6912	.6671
	ST(16)	.6540	.6845	.6582	.6810	.7251	.7392	.7144	.6823
	ST(10)	.6530	.6780	.6627	.6845	.7305	.7522	.7213	.6867
	ST(8)	.6996	.7293	.7116	.7344	.7767	.7981	.7653	.7348
	ST(7)	.7306	.7562	.7411	.7617	.8100	.8293	.7938	.7623
	ST(6)	.7884	.8147	.7890	.8181	.8696	.8863	.8539	.8177
	ST(5)	.8711	.8985	.8794	.9054	.9535	.9689	.9361	.9016
	AVG	.7182	.7468	.7256	.7515	.7966	.8153	.7854	.7529

TABLE 22

EFFICIENCIES OF ADAPTIVE ROBUST ESTIMATES OF
CANONICAL SCALE PARAMETER
(RELATIVE TO MAXIMUM LIKELIHOOD ESTIMATE IF POPULATION IS KNOWN)
(PHASE III)

SAMPLE SIZE,N	SAMPLES FROM	CRITERION (A)(1)	(A)(2)	(B)(1)	(B)(2)	(D)(1)	(D)(2)	(D)(3)	(E)
16	DS	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AS	.2079	.1941	.3227	.3165	.3451	.3475	.3248	.3185
	SB(1.5)	.9171	.9385	.9372	.9528	.9410	.9555	.9333	.9002
	SB(2.0)	.7940	.8046	.7948	.8118	.7992	.8174	.7931	.7533
	SB(2.5)	.7432	.7605	.7376	.7517	.7510	.7634	.7496	.7227
	SB(3.0)	.7217	.7462	.7115	.7204	.7275	.7449	.7172	.6903
	SB(3.5)	.6964	.7151	.6915	.7076	.7151	.7312	.7105	.6909
	SB(4.0)	.6925	.7131	.6909	.7081	.7142	.7274	.7043	.6914
	ST(16)	.6360	.6531	.6531	.6620	.7383	.7523	.7121	.6861
	ST(10)	.6643	.6806	.6769	.6870	.7597	.7768	.7309	.7010
	ST(8)	.7109	.7240	.7166	.7315	.7967	.8052	.7726	.7368
	ST(7)	.7140	.7294	.7209	.7335	.8284	.8435	.7991	.7651
	ST(6)	.7931	.8075	.8012	.8120	.9005	.9092	.8617	.8328
	ST(5)	.8909	.9073	.9132	.9236	.9930	1.0032	.9639	.9280
	AVG	.7321	.7475	.7510	.7631	.8197	.8327	.7966	.7668
20	DS	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AS	.1482	.1411	.2272	.2250	.2316	.2338	.2188	.2168
	SB(1.5)	.9450	.9468	.9688	.9783	.9632	.9745	.9486	.9173
	SB(2.0)	.8242	.8344	.8306	.8383	.8217	.8280	.8118	.7790
	SB(2.5)	.7893	.7968	.7840	.7925	.7808	.7850	.7819	.7476
	SB(3.0)	.7404	.7537	.7361	.7474	.7510	.7675	.7387	.7143
	SB(3.5)	.7437	.7550	.7283	.7351	.7477	.7607	.7406	.7105
	SB(4.0)	.7351	.7469	.7139	.7257	.7461	.7529	.7431	.7243
	ST(16)	.6464	.6561	.6639	.6697	.7630	.7716	.7150	.6858
	ST(10)	.6531	.6635	.6710	.6764	.7908	.7974	.7331	.7073
	ST(8)	.6865	.6929	.6912	.6995	.8011	.8114	.7561	.7371
	ST(7)	.7302	.7349	.7431	.7489	.8384	.8513	.7916	.7701
	ST(6)	.7697	.7757	.7785	.7876	.8772	.8868	.8246	.8047
	ST(5)	.8931	.8990	.9021	.9046	1.0052	1.0126	.9651	.9488
	AVG	.7411	.7484	.7545	.7614	.8359	.8456	.7988	.7756

TABLE 22

EFFICIENCIES OF ADAPTIVE ROBUST ESTIMATES OF
CANONICAL SCALE PARAMETER
(RELATIVE TO MAXIMUM LIKELIHOOD ESTIMATE IF POPULATION IS KNOWN)
(PHASE III)

SAMPLE SIZE,N	SAMPLES FROM	CRITERION (A)(1)	(A)(2)	(B)(1)	(B)(2)	(D)(1)	(D)(2)	(D)(3)	(E)
24	DS	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AS	.0994	.0970	.1437	.1437	.1472	.1472	.1454	.1437
	SB(1.5)	.9904	.9878	.9957	.9957	.9878	.9957	.9852	.9574
	SB(2.0)	.8847	.8774	.8720	.8756	.8544	.8596	.8493	.8198
	SB(2.5)	.8214	.8228	.8157	.8186	.8060	.8074	.7993	.7720
	SB(3.0)	.8111	.8175	.7924	.7949	.7852	.7924	.7805	.7598
	SB(3.5)	.8055	.8079	.7861	.7872	.7928	.8032	.7839	.7581
	SB(4.0)	.7858	.7913	.7624	.7655	.7782	.7858	.7676	.7433
	ST(16)	.6259	.6285	.6405	.6444	.7475	.7584	.6956	.6784
	ST(10)	.6560	.6581	.6669	.6684	.7764	.7877	.7262	.7046
	ST(8)	.6931	.6948	.7001	.7021	.7934	.8042	.7449	.7273
	ST(7)	.6976	.7004	.7123	.7142	.8147	.8222	.7504	.7400
	ST(6)	.7446	.7475	.7618	.7627	.8593	.8652	.8048	.7990
	ST(5)	.9200	.9223	.9432	.9454	1.0155	1.0270	.9722	.9619
	AVG	.7486	.7507	.7618	.7639	.8377	.8466	.7958	.7797

TABLE 23

EFFICIENCIES OF DEBIASED ADAPTIVE ROBUST ESTIMATES OF
SCALE PARAMETER
(RELATIVE TO DEBIASED MAXIMUM LIKELIHOOD ESTIMATE IF POPULATION IS KNOWN)
(PHASE III)

SAMPLE SIZE, N	SAMPLES FROM	CRITERION (A)(1)	(A)(2)	(B)(1)	(B)(2)	(C)(1)	(C)(2)	(C)(3)	(E)
6	DS	.0646	.0755	.0755	.0755	.0633	.0633	.0646	.0646
	AS	.7437	.7255	.7437	.7291	.7291	.7327	.7255	.7400
	SB(1.5)	1.1563	1.1679	1.1827	1.1888	1.1918	1.1888	1.1918	1.1858
	SB(2.0)	.9757	.9910	.9822	1.0045	1.0138	1.0068	1.0133	1.0045
	SB(2.5)	.9469	.9698	.9601	.9796	.9938	.9877	.9959	.9776
	SB(3.0)	.9127	.9423	.9306	.9492	.9685	.9632	.9685	.9526
	SB(3.5)	.9288	.9492	.9381	.9606	.9723	.9689	.9757	.9590
	SB(4.0)	.9349	.9583	.9535	.9712	.9880	.9829	.9914	.9729
	ST(16)	.9167	.9390	.9328	.9581	.9758	.9713	.9803	.9571
	ST(10)	.9000	.9292	.9148	.9356	.9594	.9517	.9692	.9422
	ST(8)	.9040	.9309	.9157	.9399	.9638	.9551	.9699	.9424
	ST(7)	.9032	.9195	.9163	.9371	.9589	.9504	.9675	.9429
	ST(6)	.9192	.9401	.9351	.9531	.9695	.9642	.9795	.9524
	ST(5)	1.0564	1.0713	1.0646	1.0813	1.0977	1.0875	1.1073	1.0789
	AVG	.8748	.9027	.8964	.9133	.9159	.9104	.9227	.9039
12	DS	.0061	.0061	.0061	.0061	.0055	.0055	.0055	.0055
	AS	.3465	.3411	.3333	.3333	.3284	.3284	.3284	.3333
	SB(1.5)	1.0789	1.0885	1.1031	1.1081	1.0933	1.1031	1.0933	1.0982
	SB(2.0)	.9644	.9783	.9819	.9927	.9927	.9963	.9891	.9891
	SB(2.5)	.9045	.9251	.9191	.9342	.9435	.9467	.9404	.9281
	SB(3.0)	.8915	.9129	.9021	.9212	.9354	.9383	.9354	.9268
	SB(3.5)	.9008	.9191	.9164	.9326	.9436	.9464	.9464	.9244
	SB(4.0)	.8990	.9180	.9156	.9303	.9429	.9455	.9429	.9278
	ST(16)	.8569	.8808	.8754	.8905	.9389	.9343	.9389	.9089
	ST(10)	.8470	.8635	.8611	.8745	.9199	.9104	.9185	.8683
	ST(8)	.9013	.9152	.9117	.9223	.9635	.9557	.9635	.9356
	ST(7)	.8868	.9029	.9029	.9129	.9502	.9430	.9502	.9231
	ST(6)	.9374	.9505	.9434	.9567	.9966	.9888	.9989	.9704
	ST(5)	1.0220	1.0344	1.0292	1.0408	1.0736	1.0669	1.0747	1.0493
	AVG	.7975	.8095	.8066	.8161	.8279	.8249	.8280	.8117

TABLE 23

EFFICIENCIES OF DEBIASED ADAPTIVE ROBUST ESTIMATES OF
SCALE PARAMETER
(RELATIVE TO DEBIASED MAXIMUM LIKELIHOOD ESTIMATE IF POPULATION IS KNOWN)
(PHASE III)

SAMPLE SIZE,N	SAMPLES FROM	CRITERION (A)(1)	(A)(2)	(B)(1)	(B)(2)	(D)(1)	(D)(2)	(D)(3)	(E)
16	DS	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AS	.1382	.1382	.1339	.1328	.1339	.1328	.1328	.1339
	SB(1.5)	1.0875	1.0943	1.1083	1.1154	1.0943	1.1083	1.1013	1.1154
	SB(2.0)	.8846	.9020	.9200	.9293	.9200	.9293	.9200	.9200
	SB(2.5)	.8452	.8670	.8821	.8938	.8860	.8978	.8860	.8860
	SB(3.0)	.8228	.8427	.8636	.8782	.8782	.8856	.8782	.8708
	SB(3.5)	.8459	.8721	.8893	.9000	.9109	.9184	.9146	.9109
	SB(4.0)	.8369	.8613	.8741	.8872	.9112	.9147	.9077	.9077
	ST(16)	.8337	.8480	.8627	.8703	.9362	.9362	.9188	.8980
	ST(10)	.8381	.8486	.8533	.8579	.9217	.9163	.9058	.8820
	ST(8)	.8559	.8646	.8661	.8750	.9291	.9274	.9158	.8887
	ST(7)	.8470	.8549	.8603	.8657	.9272	.9241	.9195	.8940
	ST(6)	.9015	.9079	.9118	.9157	.9742	.9727	.9596	.9399
	ST(5)	1.0091	1.0157	1.0251	1.0278	1.0777	1.0732	1.0615	1.0402
	AVG	.7112	.7190	.7172	.7215	.7482	.7483	.7420	.7318
20	DS	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AS	.0559	.0559	.0544	.0544	.0548	.0544	.0548	.0548
	SB(1.5)	.9302	.9449	.9524	.9524	.9449	.9524	.9449	.9524
	SB(2.0)	.7874	.8059	.8253	.8405	.8303	.8457	.8354	.8354
	SB(2.5)	.7594	.7802	.7889	.8023	.8023	.8208	.8023	.8023
	SB(3.0)	.7667	.7892	.8214	.8299	.8385	.8474	.8429	.8385
	SB(3.5)	.7957	.8133	.8433	.8551	.8673	.8756	.8632	.8551
	SB(4.0)	.8136	.8384	.8571	.8688	.8848	.8972	.8807	.8807
	ST(16)	.8020	.8082	.8272	.8316	.9054	.9054	.8753	.8564
	ST(10)	.7756	.7859	.8036	.8091	.8878	.8900	.8558	.8396
	ST(8)	.7969	.8031	.8110	.8158	.8841	.8803	.8619	.8495
	ST(7)	.8394	.8439	.8532	.8564	.9263	.9190	.8994	.8840
	ST(6)	.8628	.8659	.8719	.8750	.9395	.9360	.9086	.8955
	ST(5)	.9910	.9955	.9970	1.0000	1.0666	1.0631	1.0429	1.0330
	AVG	.6331	.6386	.6433	.6465	.6727	.6739	.6622	.6565

TABLE 23

EFFICIENCIES OF DEBIASED ADAPTIVE ROBUST ESTIMATES OF SCALE PARAMETER

(RELATIVE TO DEBIASED MAXIMUM LIKELIHOOD ESTIMATE IF POPULATION IS KNOWN)

(PHASE III)

SAMPLE SIZE,N	SAMPLES FROM	CRITERION (A)(1)	(A)(2)	(B)(1)	(B)(2)	(D)(1)	(D)(2)	(D)(3)	(E)
24	DS	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AS	.0250	.0250	.0247	.0247	.0247	.0247	.0247	.0248
	SB(1.5)	.8083	.8151	.8362	.8435	.8435	.8509	.8435	.8509
	SB(2.0)	.6752	.6883	.7260	.7361	.7413	.7518	.7413	.7413
	SB(2.5)	.6780	.6897	.7317	.7407	.7547	.7643	.7595	.7547
	SB(3.0)	.7151	.7268	.7688	.7778	.7917	.8012	.7917	.7917
	SB(3.5)	.7178	.7323	.7713	.7880	.8056	.8192	.8056	.8011
	SB(4.0)	.7400	.7513	.7914	.8000	.8268	.8315	.8268	.8222
	ST(16)	.7768	.7813	.8024	.8072	.8874	.8904	.8562	.8454
	ST(10)	.7755	.7775	.7958	.7979	.8686	.8736	.8398	.8283
	ST(8)	.7878	.7878	.8005	.8005	.8660	.8703	.8389	.8270
	ST(7)	.8043	.8060	.8184	.8202	.8863	.8863	.8500	.8442
	ST(6)	.8346	.8379	.8546	.8563	.9186	.9226	.8900	.8864
	ST(5)	1.0272	1.0290	1.0442	1.0442	1.0946	1.0967	1.0698	1.0638
	AVG	.5761	.5787	.5883	.5904	.6141	.6164	.6042	.6013

TABLE 24

EFFICIENCIES OF DEBIASED ADAPTIVE ROBUST ESTIMATES OF
CANONICAL SCALE PARAMETER
(RELATIVE TO DEBIASED MAXIMUM LIKELIHOOD ESTIMATE IF POPULATION IS KNOWN)
(PHASE III)

SAMPLE SIZE,N	SAMPLES FROM	CRITERION (A)(1)	(A)(2)	(B)(1)	(B)(2)	(D)(1)	(D)(2)	(D)(3)	(E)
8	DS	.0451	.0262	.0262	.0262	.1059	.1059	.0451	.0451
	AS	.2210	.2083	.3106	.2823	.4063	.3705	.3425	.3136
	SB(1.5)	.8606	.8915	.9033	.9374	1.0527	1.0120	1.0228	.9067
	SB(2.0)	.7886	.8138	.7865	.8244	.9122	.8867	.8915	.8043
	SB(2.5)	.7790	.8174	.7827	.8235	.9082	.8909	.8919	.8022
	SB(3.0)	.7613	.8116	.7781	.8141	.8894	.8720	.8670	.7964
	SB(3.5)	.7975	.8301	.8014	.8356	.8949	.8784	.8822	.8138
	SB(4.0)	.7813	.8218	.7980	.8318	.9051	.8945	.8849	.8169
	ST(16)	.7904	.8236	.8088	.8471	.8997	.9048	.8896	.8318
	ST(10)	.7553	.8032	.7685	.8016	.8848	.8936	.8602	.8061
	ST(8)	.7714	.8099	.7822	.8218	.8854	.8983	.8639	.8088
	ST(7)	.7684	.7932	.7815	.8098	.8827	.8897	.8602	.8089
	ST(6)	.7688	.8003	.7833	.8153	.8846	.8954	.8551	.8013
	ST(5)	.8813	.9069	.8887	.9197	.9909	1.0056	.9641	.9065
	AVG	.7579	.7726	.7609	.7895	.8925	.8927	.8541	.7959
12	DS	.0071	.0071	.0071	.0071	.0364	.0364	.0364	.0364
	AS	.1705	.1559	.3026	.2742	.3706	.3527	.3215	.2995
	SB(1.5)	.9157	.9091	.9597	.9466	1.0413	.9989	1.0245	.9258
	SB(2.0)	.8353	.8464	.8345	.8384	.9131	.8939	.8930	.8103
	SB(2.5)	.8043	.8138	.7835	.8010	.8584	.8413	.8516	.7644
	SB(3.0)	.7827	.7950	.7614	.7724	.8517	.8503	.8456	.7758
	SB(3.5)	.7541	.7851	.7552	.7770	.8533	.8507	.8387	.7472
	SB(4.0)	.7823	.8045	.7683	.7912	.8576	.8518	.8409	.7736
	ST(16)	.7164	.7411	.7222	.7404	.8327	.8418	.8148	.7546
	ST(10)	.6882	.7076	.7001	.7148	.7997	.8127	.7840	.7279
	ST(8)	.7222	.7446	.7357	.7526	.8318	.8437	.8148	.7644
	ST(7)	.7228	.7416	.7352	.7487	.8291	.8394	.8062	.7584
	ST(6)	.7618	.7803	.7639	.7857	.8627	.8734	.8440	.7949
	ST(5)	.8098	.8290	.8181	.8367	.9083	.9175	.8870	.8411
	AVG	.7282	.7449	.7383	.7531	.8484	.8532	.8300	.7727

TABLE 24

EFFICIENCIES OF DEBIASED ADAPTIVE ROBUST ESTIMATES OF CANONICAL SCALE PARAMETER (RELATIVE TO DEBIASED MAXIMUM LIKELIHOOD ESTIMATE IF POPULATION IS KNOWN) (PHASE III)

SAMPLE SIZE,N	SAMPLES FROM	CRITERION (A)(1)	(A)(2)	(B)(1)	(B)(2)	(D)(1)	(D)(2)	(D)(3)	(E)
16	DS	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AS	.1402	.1252	.3072	.2864	.3797	.3713	.3281	.3188
	SB(1.5)	1.0675	1.0467	1.1117	1.0847	1.1328	1.0914	1.1186	1.0228
	SB(2.0)	.8934	.8766	.8829	.8740	.9095	.8947	.8960	.8098
	SB(2.5)	.8313	.8293	.8098	.8041	.8528	.8424	.8486	.7858
	SB(3.0)	.7995	.8057	.7757	.7644	.8247	.8192	.8057	.7464
	SB(3.5)	.7715	.7738	.7584	.7634	.8215	.8223	.8082	.7584
	SB(4.0)	.7731	.7838	.7668	.7731	.8307	.8298	.8077	.7689
	ST(16)	.6618	.6747	.6818	.6863	.6102	.8179	.7671	.7243
	ST(10)	.6598	.6717	.6729	.6792	.7814	.7933	.7426	.7014
	ST(3)	.6846	.6937	.6883	.6982	.7918	.7961	.7574	.7116
	ST(7)	.6669	.6774	.6730	.6814	.7956	.8059	.7602	.7186
	ST(6)	.7078	.7177	.7151	.7211	.8243	.8291	.7820	.7479
	ST(5)	.7813	.7915	.8000	.8056	.8888	.8945	.8546	.8154
	AVG	.7098	.7172	.7307	.7353	.8276	.8314	.7944	.7501
20	DS	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AS	.1223	.1112	.3534	.3383	.3975	.4077	.3383	.3313
	SB(1.5)	1.0090	.9745	1.0672	1.0373	1.0520	1.0286	1.0315	.9644
	SB(2.0)	.8878	.8690	.8826	.8673	.8808	.8623	.8607	.8025
	SB(2.5)	.8165	.8030	.7964	.7899	.8138	.7990	.8097	.7496
	SB(3.0)	.7840	.7840	.7700	.7690	.8123	.8123	.7873	.7397
	SB(3.5)	.7967	.7938	.7721	.7694	.8219	.8209	.8026	.7516
	SB(4.0)	.7814	.7805	.7475	.7533	.8146	.8127	.7995	.7641
	ST(16)	.6399	.6458	.6571	.6592	.7860	.7910	.7224	.6846
	ST(10)	.6142	.6207	.6318	.6343	.7684	.7709	.7024	.6720
	ST(3)	.6194	.6229	.6239	.6289	.7382	.7452	.6901	.6685
	ST(7)	.6558	.6581	.6659	.6690	.7706	.7797	.7204	.6961
	ST(6)	.6682	.6716	.6782	.6841	.7801	.7851	.7256	.7042
	ST(5)	.7473	.7503	.7542	.7553	.8546	.8591	.8140	.7966
	AVG	.6848	.6863	.6982	.7002	.7979	.8011	.7522	.7227

TABLE 24
EFFICIENCIES OF DEBIASED ADAPTIVE ROBUST ESTIMATES OF
CANONICAL SCALE PARAMETER
(RELATIVE TO DEBIASED MAXIMUM LIKELIHOOD ESTIMATE IF POPULATION IS KNOWN)
(PHASE III)

SAMPLE SIZE,N	SAMPLES FROM	CRITERION (A)(1)	(A)(2)	(B)(1)	(B)(2)	(D)(1)	(D)(2)	(D)(3)	(E)
24	DS	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AS	.1060	.0994	.3614	.3457	.4417	.4417	.3975	.3975
	SB(1.5)	1.0729	1.0503	1.0925	1.0729	1.0729	1.0466	1.0691	1.0181
	SB(2.0)	.8856	.8659	.8745	.8659	.8469	.8348	.8408	.7948
	SB(2.5)	.8451	.8365	.8314	.8263	.8214	.8116	.8084	.7675
	SB(3.0)	.8379	.8363	.8144	.8101	.8130	.8115	.8004	.7699
	SB(3.5)	.8362	.8322	.8074	.8024	.8216	.8229	.8036	.7665
	SB(4.0)	.8120	.8120	.7833	.7810	.8120	.8120	.7903	.7544
	ST(16)	.6099	.6113	.6243	.6269	.7458	.7549	.6848	.6645
	ST(10)	.6061	.6073	.6160	.6170	.7317	.7407	.6779	.6555
	ST(8)	.6129	.6137	.6192	.6206	.7135	.7220	.6644	.6468
	ST(7)	.6112	.6127	.6234	.6247	.7230	.7294	.6607	.6500
	ST(6)	.6284	.6305	.6438	.6440	.7337	.7384	.6830	.6763
	ST(5)	.7552	.7567	.7739	.7749	.8423	.8512	.8015	.7913
	AVG	.6756	.6757	.6888	.6889	.7710	.7762	.7249	.7067

TABLE 25

CRITICAL VALUES OF CRITERIA FOR CLASSIFICATION AS U, N OR D
(DETERMINED BY FIVE-POINT LAGRANGIAN INTERPOLATION IN TABLE 1)

CRITERION	CRITICAL VALUES	N=10	N=14	N=18	N=22
(A)(1)	K_{L1}	2.0428	2.1050	2.1356	2.1467
	K_{U1}	2.4969	2.7253	2.8590	2.9668
(A)(2)	K_{L2}	1.9599	2.0518	2.0978	2.1200
	K_{U2}	2.6131	2.7822	2.8836	2.9760
(B)(1)	Q_{L1}	1.9238	2.0253	2.0934	2.1435
	Q_{U1}	2.1737	2.3907	2.5481	2.6762
(B)(2)	Q_{L2}	1.8721	1.9946	2.0715	2.1278
	Q_{U2}	2.2245	2.4120	2.5560	2.6802
(D)(1)	λ^*_{11}	.7600	.8038	.8345	.8569
	λ^*_{21}	.8316	.8741	.8907	.8915
	λ^*_{31}	1.0652	1.0423	1.0179	1.0120
(D)(2)	λ^*_{12}	.7468	.7947	.8274	.8508
	λ^*_{22}	.8616	.8957	.9148	.9275
	λ^*_{32}	1.0639	1.0432	1.0182	1.0119
(D)(3),(E)	λ^*_{13}	.7663	.8081	.8373	.8580
	λ^*_{23}	.7419	.7886	.8162	.8385
	λ^*_{33}	.9690	.9774	.9821	.9846

TABLE 26

DEBIASING FACTORS FOR MAXIMUM LIKELIHOOD ESTIMATORS OF SCALE PARAMETER FOR DOUBLE SPIKE, ARC SINE, SYMMETRIC BETA AND STUDENT T POPULATIONS
PHASE IV: N=10(4)22

POPULATION	DEBIASING FACTORS			
	N=10	N=14	N=18	N=22
DS	1.0020	1.0002	1.0000	1.0000
AS	1.0762	1.0408	1.0265	1.0162
SB(1.5)	1.2589	1.2052	1.1728	1.1404
SB(2.0)	1.1920	1.1510	1.1237	1.1046
SB(2.5)	1.1528	1.1198	1.0967	1.0765
SB(3.0)	1.1375	1.0993	1.0792	1.0710
SB(3.5)	1.1174	1.0911	1.0750	1.0546
SB(4.0)	1.1234	1.0860	1.0665	1.0541
ST(16)	1.0546	1.0310	1.0170	1.0052
ST(10)	1.0374	1.0156	1.0037	.9913
ST(8)	1.0331	1.0091	.9932	.9815
ST(7)	1.0237	.9958	.9861	.9793
ST(6)	1.0211	.9915	.9785	.9736
ST(5)	1.0053	.9818	.9713	.9548

TABLE 27

CONTINGENCY TABLES--CLASSIFICATION VS. TRUE POPULATION BY CRITERIA
(PHASE IV: N=10)

		CRITERION (A)(1)				CRITERION (A)(2)			
		CLASSIFIED AS				CLASSIFIED AS			
		U	N	D	SUMS	U	N	D	SUMS
NUMBER OF	U	3247	1051	702	5000	2895	1537	568	5000
SAMPLES FROM	N	1654	1437	1909	5000	1366	2018	1616	5000
	D	784	1030	3186	5000	599	1466	2935	5000
	DS	4481	0	519	5000	4469	4	527	5000
	AS	3843	662	495	5000	3644	957	399	5000
	SB(1.5)	2835	1287	878	5000	2510	1785	705	5000
	SB(2.0)	2590	1327	1083	5000	2250	1878	872	5000
	SB(2.5)	2334	1490	1176	5000	2014	2029	957	5000
	SB(3.0)	2235	1424	1341	5000	1927	1973	1100	5000
	SB(3.5)	2152	1450	1398	5000	1808	2078	1114	5000
	SB(4.0)	2049	1474	1477	5000	1730	2050	1220	5000
	ST(16)	1554	1353	2093	5000	1279	1891	1830	5000
	ST(10)	1367	1280	2353	5000	1103	1836	2061	5000
	ST(8)	1382	1289	2329	5000	1127	1810	2063	5000
	ST(7)	1277	1282	2441	5000	1034	1793	2173	5000
	ST(6)	1229	1211	2560	5000	1018	1710	2272	5000
	ST(5)	1145	1260	2595	5000	938	1726	2336	5000
	SUMS	36158	20307	28535	85000	31711	28541	24748	85000

		CRITERION (B)(1)				CRITERION (B)(2)			
NUMBER OF	U	3328	1037	635	5000	2988	1511	501	5000
SAMPLES FROM	N	1637	1438	1925	5000	1356	1975	1669	5000
	D	799	1024	3177	5000	622	1424	2954	5000
	DS	4468	3	529	5000	4468	3	529	5000
	AS	4171	553	276	5000	3981	798	221	5000
	SB(1.5)	2839	1260	901	5000	2489	1790	721	5000
	SB(2.0)	2537	1320	1143	5000	2198	1851	951	5000
	SB(2.5)	2329	1441	1230	5000	1963	2020	1017	5000
	SB(3.0)	2216	1420	1364	5000	1888	1965	1147	5000
	SB(3.5)	2110	1464	1426	5000	1769	2045	1186	5000
	SB(4.0)	2044	1441	1515	5000	1717	2038	1245	5000
	ST(16)	1543	1334	2123	5000	1285	1833	1882	5000
	ST(10)	1357	1326	2317	5000	1090	1840	2070	5000
	ST(8)	1359	1303	2338	5000	1092	1816	2092	5000
	ST(7)	1283	1303	2414	5000	1040	1798	2162	5000
	ST(6)	1238	1265	2497	5000	1007	1737	2256	5000
	ST(5)	1145	1223	2632	5000	923	1708	2369	5000
	SUMS	36403	20155	28442	85000	31876	28152	24972	85000

TABLE 27

CONTINGENCY TABLES--CLASSIFICATION VS. TRUE POPULATION BY CRITERIA
(PHASE IV: N=10)

		CRITERION (D)(1)				CRITERION (D)(2)			
		CLASSIFIED AS				CLASSIFIED AS			
		U	N	D	SUMS	U	N	D	SUMS
NUMBER OF SAMPLES FROM	U	3306	1379	315	5000	3026	1712	262	5000
	N	1530	2423	1047	5000	1376	2679	945	5000
	D	883	1770	2347	5000	905	1912	2183	5000
	DS	4469	8	523	5000	4469	8	523	5000
	AS	4340	501	159	5000	4196	678	126	5000
	SB(1.5)	2714	1849	437	5000	2408	2230	362	5000
	SB(2.0)	2386	2054	560	5000	2156	2372	472	5000
	SB(2.5)	2193	2216	591	5000	1942	2551	507	5000
	SB(3.0)	2052	2243	705	5000	1806	2577	617	5000
	SB(3.5)	1952	2317	731	5000	1746	2631	623	5000
	SB(4.0)	1915	2309	776	5000	1715	2605	680	5000
	ST(16)	1487	2278	1235	5000	1370	2501	1129	5000
	ST(10)	1291	2277	1432	5000	1201	2499	1300	5000
	ST(8)	1340	2205	1455	5000	1239	2426	1335	5000
	ST(7)	1267	2213	1520	5000	1182	2419	1399	5000
	ST(6)	1231	2170	1599	5000	1155	2370	1475	5000
	ST(5)	1136	2130	1734	5000	1097	2300	1603	5000
	SUMS	35492	32342	17166	85000	32989	36470	15541	85000

		CRITERION (D)(3)				CRITERION (E)			
NUMBER OF SAMPLES FROM	U	3284	1252	464	5000	3258	1008	734	5000
	N	1509	2290	1201	5000	1495	1730	1775	5000
	D	770	1694	2536	5000	757	1134	3109	5000
	DS	4469	8	523	5000	4469	3	528	5000
	AS	4273	437	290	5000	4260	337	403	5000
	SB(1.5)	2749	1680	571	5000	2728	1344	928	5000
	SB(2.0)	2385	1918	697	5000	2364	1520	1116	5000
	SB(2.5)	2199	2056	745	5000	2167	1615	1218	5000
	SB(3.0)	2064	2092	844	5000	2045	1652	1303	5000
	SB(3.5)	1964	2166	870	5000	1947	1715	1338	5000
	SB(4.0)	1874	2186	940	5000	1853	1724	1423	5000
	ST(16)	1427	2172	1401	5000	1406	1577	2017	5000
	ST(10)	1249	2158	1593	5000	1234	1578	2188	5000
	ST(8)	1280	2109	1611	5000	1270	1537	2193	5000
	ST(7)	1189	2120	1691	5000	1176	1519	2305	5000
	ST(6)	1169	2077	1754	5000	1146	1483	2371	5000
	ST(5)	1067	2033	1900	5000	1055	1460	2485	5000
	SUMS	34921	30448	19631	85000	34630	22936	27434	85000

TABLE 28

CONTINGENCY TABLES--CLASSIFICATION VS. TRUE POPULATION BY CRITERIA
(PHASE IV: N=14)

		CRITERION (A)(1)				CRITERION (A)(2)			
		CLASSIFIED AS				CLASSIFIED AS			
		U	N	D	SUMS	U	N	D	SUMS
NUMBER OF	U	3566	1133	301	5000	3356	1382	262	5000
SAMPLES FROM	N	1443	1906	1651	5000	1261	2194	1545	5000
	D	481	1246	3273	5000	402	1452	3146	5000
	DS	4722	0	278	5000	4721	0	279	5000
	AS	4317	508	175	5000	4213	636	151	5000
	SB(1.5)	3026	1502	472	5000	2796	1794	410	5000
	SB(2.0)	2619	1759	622	5000	2384	2068	548	5000
	SB(2.5)	2351	1871	778	5000	2131	2200	669	5000
	SB(3.0)	2177	1952	871	5000	1961	2252	787	5000
	SB(3.5)	2077	1935	988	5000	1804	2313	883	5000
	SB(4.0)	2010	1894	1096	5000	1766	2246	988	5000
	ST(16)	1147	1915	1938	5000	990	2208	1802	5000
	ST(10)	1057	1704	2239	5000	923	1962	2115	5000
	ST(8)	1025	1621	2354	5000	880	1882	2238	5000
	ST(7)	958	1597	2445	5000	816	1857	2327	5000
	ST(6)	942	1527	2531	5000	824	1771	2405	5000
	ST(5)	773	1509	2718	5000	662	1744	2594	5000
	SUMS	34691	25579	24730	85000	31890	29961	23149	85000

		CRITERION (B)(1)				CRITERION (B)(2)			
NUMBER OF	U	3626	1131	243	5000	3452	1339	209	5000
SAMPLES FROM	N	1309	2028	1663	5000	1155	2278	1567	5000
	D	476	1228	3296	5000	389	1393	3218	5000
	DS	4721	221	58	5000	4721	221	58	5000
	AS	4612	325	63	5000	4550	390	60	5000
	SB(1.5)	2918	1628	454	5000	2715	1877	408	5000
	SB(2.0)	2506	1855	639	5000	2277	2135	588	5000
	SB(2.5)	2260	1946	794	5000	2026	2245	729	5000
	SB(3.0)	2088	1986	926	5000	1862	2272	866	5000
	SB(3.5)	1938	2041	1021	5000	1737	2305	958	5000
	SB(4.0)	1879	1983	1138	5000	1691	2239	1070	5000
	ST(16)	1101	1973	1926	5000	956	2220	1824	5000
	ST(10)	1008	1759	2233	5000	860	2013	2127	5000
	ST(8)	1014	1688	2298	5000	889	1894	2217	5000
	ST(7)	885	1679	2436	5000	773	1896	2331	5000
	ST(6)	886	1641	2473	5000	776	1848	2376	5000
	ST(5)	746	1584	2670	5000	638	1781	2581	5000
	SUMS	33973	26696	24331	85000	31467	30346	23187	85000

TABLE 28

CONTINGENCY TABLES--CLASSIFICATION VS. TRUE POPULATION BY CRITERIA
(PHASE IV: N=14)

		CRITERION (D)(1)				CRITERION (D)(2)			
		CLASSIFIED AS				CLASSIFIED AS			
		U	N	D	SUMS	U	N	D	SUMS
NUMBER OF SAMPLES FROM	U	3625	1245	130	5000	3454	1441	105	5000
	N	1269	2827	904	5000	1148	2996	856	5000
	D	567	1891	2542	5000	557	1970	2473	5000
	OS	4942	1	57	5000	4942	1	57	5000
	AS	4726	219	55	5000	4699	256	45	5000
	SB(1.5)	2840	1915	245	5000	2629	2154	217	5000
	SB(2.0)	2387	2276	337	5000	2168	2528	304	5000
	SB(2.5)	2113	2507	380	5000	1939	2716	345	5000
	SB(3.0)	1938	2573	489	5000	1775	2780	445	5000
	SB(3.5)	1809	2707	484	5000	1668	2889	443	5000
	SB(4.0)	1737	2661	602	5000	1578	2875	547	5000
	ST(16)	1090	2776	1134	5000	998	2935	1067	5000
	ST(10)	984	2684	1332	5000	898	2829	1273	5000
	ST(8)	1017	2527	1456	5000	913	2675	1412	5000
	ST(7)	917	2513	1570	5000	878	2628	1494	5000
	ST(6)	888	2491	1621	5000	821	2616	1563	5000
	ST(5)	754	2454	1792	5000	704	2567	1729	5000
	SUMS	33603	36267	15130	85000	31769	38856	14375	85000

		CRITERION (D)(3)				CRITERION (E)			
NUMBER OF SAMPLES FROM	U	3611	1159	230	5000	3601	1030	369	5000
	N	1258	2605	1137	5000	1255	2149	1596	5000
	D	500	1653	2847	5000	490	1229	3281	5000
	OS	4721	1	278	5000	4721	1	278	5000
	AS	4705	191	104	5000	4700	157	143	5000
	SB(1.5)	2847	1798	355	5000	2840	1594	566	5000
	SB(2.0)	2424	2134	442	5000	2412	1861	727	5000
	SB(2.5)	2146	2339	515	5000	2139	2024	837	5000
	SB(3.0)	1926	2429	645	5000	1918	2116	966	5000
	SB(3.5)	1802	2544	654	5000	1793	2165	1042	5000
	SB(4.0)	1781	2463	756	5000	1769	2128	1103	5000
	ST(16)	1062	2544	1394	5000	1048	2079	1873	5000
	ST(10)	946	2447	1607	5000	932	1945	2123	5000
	ST(8)	957	2285	1758	5000	953	1842	2205	5000
	ST(7)	863	2278	1859	5000	854	1821	2325	5000
	ST(6)	841	2242	1917	5000	833	1793	2374	5000
	ST(5)	741	2167	2092	5000	733	1728	2539	5000
	SUMS	33131	33279	18590	85000	32991	27662	24347	85000

TABLE 29

CONTINGENCY TABLES--CLASSIFICATION VS. TRUE POPULATION BY CRITERIA
(PHASE IV: N=18)

		CRITERION (A)(1)				CRITERION (A)(2)			
		CLASSIFIED AS				CLASSIFIED AS			
		U	N	D	SUMS	U	N	D	SUMS
NUMBER OF	U	3858	1026	116	5000	3724	1165	111	5000
SAMPLES FROM	N	1228	2182	1590	5000	1103	2362	1535	5000
	D	237	1288	3475	5000	204	1376	3420	5000
	DS	4818	134	48	5000	4818	134	48	5000
	AS	4560	391	49	5000	4509	443	48	5000
	SB(1.5)	3095	1617	288	5000	2946	1785	269	5000
	SB(2.0)	2684	1943	373	5000	2486	2167	347	5000
	SB(2.5)	2212	2192	596	5000	2045	2389	566	5000
	SB(3.0)	2085	2215	700	5000	1905	2439	656	5000
	SB(3.5)	1959	2277	764	5000	1775	2497	728	5000
	SB(4.0)	1860	2302	838	5000	1695	2512	793	5000
	ST(16)	939	2113	1948	5000	837	2266	1897	5000
	ST(10)	821	2043	2136	5000	735	2186	2079	5000
	ST(8)	759	1861	2380	5000	671	1995	2334	5000
	ST(7)	669	1843	2488	5000	583	1971	2446	5000
	ST(6)	621	1767	2612	5000	534	1916	2550	5000
	ST(5)	542	1633	2825	5000	473	1763	2764	5000
	SUMS	32947	28827	23226	85000	31043	31366	22591	85000

		CRITERION (B)(1)				CRITERION (B)(2)			
NUMBER OF	U	3912	1003	85	5000	3804	1111	85	5000
SAMPLES FROM	N	1104	2315	1581	5000	982	2465	1553	5000
	D	237	1288	3475	5000	197	1356	3447	5000
	DS	4818	134	48	5000	4818	134	48	5000
	AS	4827	166	7	5000	4804	190	6	5000
	SB(1.5)	3010	1728	262	5000	2863	1886	251	5000
	SB(2.0)	2516	2099	385	5000	2362	2270	368	5000
	SB(2.5)	2084	2309	607	5000	1942	2465	593	5000
	SB(3.0)	1898	2393	709	5000	1743	2567	690	5000
	SB(3.5)	1803	2363	834	5000	1655	2535	810	5000
	SB(4.0)	1674	2440	886	5000	1525	2624	851	5000
	ST(16)	852	2204	1944	5000	773	2325	1902	5000
	ST(10)	759	2145	2096	5000	678	2267	2055	5000
	ST(8)	686	1944	2370	5000	623	2042	2335	5000
	ST(7)	616	1939	2445	5000	552	2038	2410	5000
	ST(6)	583	1880	2537	5000	500	1996	2504	5000
	ST(5)	503	1760	2737	5000	458	1842	2700	5000
	SUMS	31882	30110	23008	85000	30279	32113	22608	85000

TABLE 29

CONTINGENCY TABLES--CLASSIFICATION VS. TRUE POPULATION BY CRITERIA
(PHASE IV: N=18)

		CRITERION (D)(1) CLASSIFIED AS				CRITERION (D)(2) CLASSIFIED AS			
		U	N	D	SUMS	U	N	D	SUMS
NUMBER OF	U	3943	990	67	5000	3814	1131	55	5000
SAMPLES FROM	N	1029	3073	898	5000	943	3200	857	5000
	D	328	1772	2900	5000	341	1818	2841	5000
	DS	4952	0	48	5000	4952	0	48	5000
	AS	4900	92	8	5000	4883	110	7	5000
	SB(1.5)	2966	1870	164	5000	2810	2044	146	5000
	SB(2.0)	2409	2358	233	5000	2248	2547	205	5000
	SB(2.5)	1985	2683	332	5000	1862	2831	307	5000
	SB(3.0)	1779	2842	379	5000	1656	2995	349	5000
	SB(3.5)	1687	2902	411	5000	1554	3062	384	5000
	SB(4.0)	1550	2990	460	5000	1422	3142	436	5000
	ST(16)	818	3036	1146	5000	769	3119	1112	5000
	ST(10)	749	2953	1298	5000	688	3049	1263	5000
	ST(8)	686	2830	1484	5000	632	2926	1442	5000
	ST(7)	619	2729	1652	5000	581	2801	1618	5000
	ST(6)	577	2664	1759	5000	563	2732	1705	5000
	ST(5)	539	2506	1955	5000	507	2574	1919	5000
	SUMS	31516	38290	15194	85000	30225	40081	14694	85000

		CRITERION (D)(3)				CRITERION (E)			
NUMBER OF	U	3954	937	109	5000	3949	855	196	5000
SAMPLES FROM	N	1032	2776	1192	5000	1026	2475	1499	5000
	D	282	1456	3262	5000	280	1201	3519	5000
	DS	4818	0	182	5000	4818	0	182	5000
	AS	4888	83	29	5000	4885	75	40	5000
	SB(1.5)	2991	1753	256	5000	2979	1622	399	5000
	SB(2.0)	2442	2248	310	5000	2436	2081	483	5000
	SB(2.5)	2022	2526	452	5000	2007	2316	677	5000
	SB(3.0)	1796	2662	542	5000	1791	2430	779	5000
	SB(3.5)	1712	2701	587	5000	1702	2477	821	5000
	SB(4.0)	1572	2797	631	5000	1566	2559	875	5000
	ST(16)	816	2678	1506	5000	812	2394	1794	5000
	ST(10)	738	2598	1664	5000	730	2309	1961	5000
	ST(8)	696	2416	1888	5000	688	2109	2203	5000
	ST(7)	611	2357	2032	5000	603	2065	2332	5000
	ST(6)	554	2289	2157	5000	546	2029	2425	5000
	ST(5)	525	2130	2345	5000	520	1834	2646	5000
	SUMS	31449	34407	19144	85000	31338	30831	22831	85000

TABLE 30

CONTINGENCY TABLES--CLASSIFICATION VS. TRUE POPULATION BY CRITERIA
(PHASE IV: N=22)

		CRITERION (A)(1)				CRITERION (A)(2)			
		CLASSIFIED AS				CLASSIFIED AS			
		U	N	D	SUMS	U	N	D	SUMS
NUMBER OF SAMPLES FROM	U	4049	887	64	5000	3961	981	58	5000
	N	1005	2548	1447	5000	920	2650	1430	5000
	D	162	1240	3598	5000	136	1282	3582	5000
	DS	4907	69	24	5000	4907	69	24	5000
	AS	4730	250	20	5000	4702	278	20	5000
	SB(1.5)	3247	1616	137	5000	3119	1745	136	5000
	SB(2.0)	2674	2095	231	5000	2532	2246	222	5000
	SB(2.5)	2284	2402	314	5000	2145	2548	307	5000
	SB(3.0)	1994	2570	436	5000	1848	2728	424	5000
	SB(3.5)	1804	2606	590	5000	1667	2752	581	5000
	SB(4.0)	1649	2740	611	5000	1530	2872	598	5000
	ST(16)	749	2389	1862	5000	675	2481	1844	5000
	ST(10)	606	2206	2188	5000	564	2261	2175	5000
	ST(8)	543	2064	2393	5000	490	2138	2372	5000
	ST(7)	474	2017	2509	5000	427	2085	2488	5000
	ST(6)	430	1861	2709	5000	390	1918	2692	5000
	ST(5)	396	1720	2884	5000	352	1775	2873	5000
	SUMS	31703	31280	22017	85000	30365	32809	21826	85000

		CRITERION (B)(1)				CRITERION (B)(2)			
NUMBER OF SAMPLES FROM	U	4155	825	20	5000	4087	894	19	5000
	N	897	2686	1417	5000	850	2746	1404	5000
	D	164	1227	3609	5000	149	1256	3595	5000
	DS	4907	69	24	5000	4907	69	24	5000
	AS	4936	62	2	5000	4922	76	2	5000
	SB(1.5)	3160	1711	129	5000	3051	1825	124	5000
	SB(2.0)	2544	2213	243	5000	2413	2349	238	5000
	SB(2.5)	2120	2526	354	5000	2000	2657	343	5000
	SB(3.0)	1821	2718	461	5000	1698	2854	448	5000
	SB(3.5)	1613	2754	633	5000	1514	2859	627	5000
	SB(4.0)	1496	2840	664	5000	1391	2956	653	5000
	ST(16)	664	2510	1826	5000	612	2584	1804	5000
	ST(10)	566	2304	2130	5000	522	2369	2109	5000
	ST(8)	495	2184	2321	5000	448	2247	2305	5000
	ST(7)	442	2090	2468	5000	412	2134	2454	5000
	ST(6)	397	1935	2668	5000	362	1985	2653	5000
	ST(5)	355	1857	2788	5000	328	1908	2764	5000
	SUMS	30732	32511	21757	85000	29666	33768	21566	85000

TABLE 30

CONTINGENCY TABLES--CLASSIFICATION VS. TRUE POPULATION BY CRITERIA
(PHASE IV: N=22)

		CRITERION (D)(1) CLASSIFIED AS				CRITERION (D)(2) CLASSIFIED AS			
		U	N	D	SUMS	U	N	D	SUMS
NUMBER OF SAMPLES FROM	U	4229	731	40	5000	4161	818	21	5000
	N	825	3375	800	5000	758	3470	772	5000
	D	194	1656	3150	5000	213	1675	3112	5000
	DS	4976	0	24	5000	4976	0	24	5000
	AS	4962	32	6	5000	4957	38	5	5000
	SB(1.5)	3077	1801	122	5000	2961	1931	108	5000
	SB(2.0)	2401	2427	172	5000	2264	2589	147	5000
	SB(2.5)	1986	2794	220	5000	1855	2947	198	5000
	SB(3.0)	1678	3046	276	5000	1573	3178	249	5000
	SB(3.5)	1481	3179	340	5000	1366	3314	320	5000
	SB(4.0)	1395	3244	361	5000	1296	3367	337	5000
	ST(16)	640	3292	1068	5000	589	3364	1047	5000
	ST(10)	542	3079	1379	5000	519	3125	1356	5000
	ST(8)	489	2970	1541	5000	452	3033	1515	5000
	ST(7)	460	2884	1656	5000	427	2941	1632	5000
	ST(6)	390	2696	1914	5000	382	2732	1886	5000
	ST(5)	374	2589	2037	5000	355	2635	2010	5000
	SUMS	30099	39795	15106	85000	29104	41157	14739	85000

		CRITERION (D)(3)				CRITERION (E)			
NUMBER OF SAMPLES FROM	U	4224	716	60	5000	4217	687	96	5000
	N	824	3044	1132	5000	819	2869	1312	5000
	D	185	1276	3539	5000	179	1126	3695	5000
	DS	4976	0	24	5000	4976	0	24	5000
	AS	4959	32	9	5000	4957	30	13	5000
	SB(1.5)	3091	1752	157	5000	3089	1679	232	5000
	SB(2.0)	2414	2346	240	5000	2412	2231	357	5000
	SB(2.5)	2000	2679	321	5000	1997	2558	445	5000
	SB(3.0)	1690	2905	405	5000	1684	2774	542	5000
	SB(3.5)	1498	2990	512	5000	1494	2836	670	5000
	SB(4.0)	1408	3060	532	5000	1402	2916	682	5000
	ST(16)	639	2843	1518	5000	635	2646	1719	5000
	ST(10)	539	2660	1801	5000	536	2490	1974	5000
	ST(8)	484	2522	1994	5000	481	2332	2187	5000
	ST(7)	456	2438	2106	5000	455	2261	2284	5000
	ST(6)	395	2254	2351	5000	391	2081	2528	5000
	ST(5)	366	2118	2516	5000	364	1968	2668	5000
	SUMS	30148	35635	19217	85000	30088	33484	21428	85000

TABLE 31

MEAN SQUARE ERRORS OF PARAMETER ESTIMATES (PHASE IV) IF POPULATION IS KNOWN

SAMPLE SIZE, N	POPULATION	MSE($\hat{\mu}$)	MSE($\hat{\sigma}$)	MSE($F\hat{\sigma}$)	MSE($\bar{\sigma}$)	MSE($F\bar{\sigma}$)
10	U	.0465	.0460	.1246	.0184	.0499
	N	.0987	.0546	.2097	.0566	.2174
	D	.0715	.0963	.4320	.1038	.4657
	DS	.0028	.0028	.0028	.0028	.0028
	AS	.0155	.0115	.0229	.0075	.0150
	SB(1.5)	.0680	.0453	.1397	.0335	.1033
	SB(2.0)	.0791	.0433	.1425	.0336	.1107
	SB(2.5)	.0871	.0432	.1476	.0364	.1242
	SB(3.0)	.0896	.0428	.1497	.0381	.1331
	SB(3.5)	.0905	.0449	.1597	.0415	.1474
	SB(4.0)	.0945	.0455	.1635	.0431	.1549
	ST(16)	.1088	.0614	.2414	.0652	.2562
	ST(10)	.1110	.0702	.2787	.0742	.2947
	ST(8)	.1117	.0773	.3082	.0817	.3258
	ST(7)	.1145	.0893	.3566	.0930	.3714
	ST(6)	.1150	.0905	.3611	.0936	.3736
	ST(5)	.1170	.1068	.4234	.1077	.4271
	AVG	.0836	.0572	.2155	.0547	.2102
14	U	.0246	.0251	.0681	.0098	.0266
	N	.0722	.0369	.1417	.0378	.1451
	D	.0484	.0711	.3189	.0751	.3372
	DS	.0002	.0002	.0002	.0002	.0002
	AS	.0063	.0045	.0090	.0031	.0061
	SB(1.5)	.0407	.0271	.0837	.0208	.0643
	SB(2.0)	.0532	.0271	.0892	.0226	.0743
	SB(2.5)	.0584	.0281	.0959	.0244	.0835
	SB(3.0)	.0617	.0277	.0969	.0246	.0862
	SB(3.5)	.0652	.0290	.1031	.0268	.0952
	SB(4.0)	.0658	.0290	.1044	.0272	.0977
	ST(16)	.0769	.0442	.1738	.0460	.1809
	ST(10)	.0784	.0544	.2161	.0564	.2239
	ST(8)	.0812	.0569	.2270	.0579	.2308
	ST(7)	.0821	.0648	.2589	.0643	.2568
	ST(6)	.0815	.0747	.2980	.0734	.2928
	ST(5)	.0848	.0917	.3634	.0881	.3493
	AVG	.0577	.0407	.1558	.0387	.1501

TABLE 31

MEAN SQUARE ERRORS OF PARAMETER ESTIMATES (PHASE IV) IF POPULATION IS KNOWN

SAMPLE SIZE,N	POPULATION	MSE($\hat{\mu}$)	MSE($\hat{\sigma}$)	MSE($F\hat{\sigma}$)	MSE($\bar{\sigma}$)	MSE($F\bar{\sigma}$)
18	U	.0161	.0159	.0431	.0059	.0159
	N	.0542	.0290	.1115	.0297	.1140
	D	.0385	.0544	.2439	.0573	.2573
	DS	0.0000	0.0000	0.0000	0.0000	0.0000
	AS	.0026	.0018	.0036	.0012	.0025
	SB(1.5)	.0302	.0183	.0566	.0154	.0474
	SB(2.0)	.0389	.0184	.0605	.0150	.0493
	SB(2.5)	.0462	.0194	.0663	.0168	.0572
	SB(3.0)	.0468	.0205	.0716	.0183	.0639
	SB(3.5)	.0487	.0213	.0758	.0199	.0707
	SB(4.0)	.0492	.0222	.0798	.0209	.0753
	ST(16)	.0573	.0354	.1392	.0365	.1435
	ST(10)	.0603	.0402	.1598	.0405	.1609
	ST(8)	.0609	.0466	.1860	.0459	.1832
	ST(7)	.0650	.0522	.2087	.0506	.2020
	ST(6)	.0648	.0565	.2255	.0538	.2146
	ST(5)	.0669	.0763	.3024	.0708	.2808
	AVG	.0439	.0311	.1197	.0293	.1140
22	U	.0112	.0109	.0294	.0039	.0106
	N	.0448	.0239	.0918	.0240	.0921
	D	.0291	.0453	.2032	.0468	.2098
	DS	0.0000	0.0000	0.0000	0.0000	0.0000
	AS	.0012	.0009	.0017	.0006	.0012
	SB(1.5)	.0220	.0136	.0419	.0102	.0315
	SB(2.0)	.0315	.0145	.0478	.0113	.0371
	SB(2.5)	.0359	.0149	.0508	.0128	.0436
	SB(3.0)	.0388	.0162	.0567	.0149	.0520
	SB(3.5)	.0402	.0172	.0610	.0161	.0574
	SB(4.0)	.0422	.0176	.0634	.0168	.0605
	ST(16)	.0471	.0281	.1106	.0284	.1115
	ST(10)	.0510	.0331	.1314	.0325	.1293
	ST(8)	.0515	.0390	.1556	.0374	.1492
	ST(7)	.0534	.0438	.1750	.0414	.1653
	ST(6)	.0531	.0516	.2058	.0480	.1914
	ST(5)	.0528	.0658	.2611	.0591	.2342
	AVG	.0356	.0257	.0992	.0238	.0927

TABLE 32

EFFICIENCIES OF ADAPTIVE ROBUST ESTIMATES OF LOCATION PARAMETER (RELATIVE TO MAXIMUM LIKELIHOOD ESTIMATE IF POPULATION IS KNOWN) (PHASE IV)

SAMPLE SIZE,N	SAMPLES FROM	CRITERION (A)(1)	(A)(2)	(B)(1)	(B)(2)	(D)(1)	(D)(2)	(D)(3)	(E)
10	U	.4367	.4361	.5199	.5106	.5848	.5800	.5145	.4844
	N	.8123	.8411	.8002	.8303	.8131	.8085	.8104	.7831
	D	.8358	.8510	.7734	.7961	.6499	.6012	.7529	.7716
	DS	.0263	.0263	.0263	.0263	.0263	.0263	.0263	.0263
	AS	.1620	.1652	.2586	.2456	.3603	.3613	.2500	.2393
	SB(1.5)	.6140	.6234	.6545	.6694	.7161	.7272	.6550	.6241
	SB(2.0)	.6814	.7006	.7050	.7185	.7553	.7658	.7161	.6791
	SB(2.5)	.7162	.7383	.7337	.7551	.7818	.7932	.7502	.7072
	SB(3.0)	.7384	.7639	.7481	.7715	.7790	.7931	.7567	.7265
	SB(3.5)	.7405	.7737	.7525	.7813	.7855	.8143	.7655	.7303
	SB(4.0)	.7613	.7800	.7643	.7917	.8006	.8097	.7813	.7491
	ST(16)	.8567	.8828	.8224	.8435	.7974	.7935	.8303	.8106
	ST(10)	.8748	.9023	.8401	.8640	.7884	.7705	.8436	.8318
	ST(8)	.8859	.9205	.8537	.8857	.8160	.8017	.8569	.8475
	ST(7)	.8975	.9267	.8747	.9095	.8254	.8116	.8780	.8619
	ST(6)	.9336	.9640	.8933	.9296	.8449	.8165	.8893	.8907
	ST(5)	.9894	1.0132	.9500	.9798	.8793	.8287	.9396	.9313
	AVG	.7176	.7368	.7297	.7481	.7321	.7247	.7325	.7125
14	U	.4185	.4116	.5104	.5047	.5656	.5606	.4992	.4767
	N	.7969	.8072	.7939	.8118	.8137	.8224	.8059	.7792
	D	.8309	.8435	.7841	.7987	.6777	.6450	.7455	.7585
	DS	.0036	.0036	.0077	.0077	.0172	.0172	.0036	.0036
	AS	.1332	.1321	.2507	.2373	.3364	.3419	.2553	.2441
	SB(1.5)	.5743	.5726	.6240	.6237	.6570	.6624	.6021	.5763
	SB(2.0)	.6646	.6747	.6924	.6971	.7186	.7306	.6852	.6537
	SB(2.5)	.7128	.7233	.7272	.7365	.7558	.7708	.7329	.7036
	SB(3.0)	.7362	.7501	.7435	.7571	.7688	.7754	.7350	.7118
	SB(3.5)	.7412	.7549	.7373	.7486	.7727	.7798	.7594	.7268
	SB(4.0)	.7428	.7621	.7449	.7647	.7739	.7932	.7596	.7303
	ST(16)	.8536	.8718	.8115	.8347	.8049	.8071	.8113	.7958
	ST(10)	.8591	.8742	.8164	.8435	.8008	.7983	.8250	.8188
	ST(8)	.8946	.9060	.8526	.8794	.8154	.8133	.8579	.8432
	ST(7)	.8855	.9030	.8578	.8781	.8110	.7983	.8560	.8464
	ST(6)	.9480	.9657	.8907	.9128	.8339	.8193	.8745	.8728
	ST(5)	.9826	.9995	.9229	.9497	.8690	.8501	.9082	.9038
	AVG	.7323	.7420	.7557	.7695	.7634	.7621	.7388	.7199

TABLE 32

EFFICIENCIES OF ADAPTIVE ROBUST ESTIMATES OF
LOCATION PARAMETER
(RELATIVE TO MAXIMUM LIKELIHOOD ESTIMATE IF POPULATION IS KNOWN)
(PHASE IV)

SAMPLE SIZE,N	SAMPLES FROM	CRITERION (A)(1)	(A)(2)	(B)(1)	(B)(2)	(D)(1)	(D)(2)	(D)(3)	(E)
18	U	.4461	.4317	.5479	.5267	.5746	.5627	.5452	.5160
	N	.8075	.8148	.7957	.8155	.8078	.8181	.7866	.7726
	D	.8809	.8827	.8275	.8370	.7695	.7165	.8255	.8520
	DS	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AS	.1160	.1112	.2963	.2760	.4738	.4494	.3400	.3199
	SB(1.5)	.5798	.5727	.6264	.6146	.6517	.6459	.6162	.5916
	SB(2.0)	.6640	.6639	.6893	.6909	.6892	.6901	.6718	.6536
	SB(2.5)	.6776	.6818	.7023	.7069	.7358	.7363	.7071	.6792
	SB(3.0)	.7179	.7244	.7218	.7301	.7501	.7603	.7334	.7066
	SB(3.5)	.7250	.7347	.7281	.7342	.7498	.7645	.7348	.7167
	SB(4.0)	.7537	.7715	.7607	.7763	.7813	.8002	.7720	.7468
	ST(16)	.8090	.8166	.7921	.8027	.7991	.7986	.7990	.7882
	ST(10)	.8343	.8453	.8098	.8249	.8089	.8069	.8165	.8093
	ST(8)	.8718	.8915	.8482	.8623	.8448	.8369	.8420	.8392
	ST(7)	.9078	.9178	.8702	.8785	.8633	.8475	.8764	.8675
	ST(6)	.9566	.9683	.9258	.9455	.9068	.8890	.9311	.9429
	ST(5)	1.0277	1.0379	.9788	.9926	.9542	.9429	.9745	.9793
	AVG	.7610	.7662	.7703	.7778	.7847	.7806	.7576	.7455
22	U	.4027	.3975	.5023	.4882	.5336	.5464	.5072	.4931
	N	.8160	.8251	.8084	.8151	.8376	.8385	.8277	.8178
	D	.8413	.8488	.7978	.7960	.7608	.7121	.7892	.8052
	DS	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AS	.0914	.0867	.3187	.2999	.4713	.4586	.3795	.3615
	SB(1.5)	.5638	.5572	.6088	.6030	.6032	.5983	.5936	.5792
	SB(2.0)	.6726	.6721	.6954	.6946	.6819	.6924	.6703	.6546
	SB(2.5)	.7004	.6989	.7048	.7076	.7077	.7115	.6994	.6804
	SB(3.0)	.7472	.7472	.7462	.7548	.7560	.7687	.7453	.7264
	SB(3.5)	.7480	.7533	.7445	.7476	.7673	.7728	.7508	.7323
	SB(4.0)	.7940	.7998	.7778	.7854	.7886	.7953	.7757	.7671
	ST(16)	.8682	.8724	.8451	.8537	.8539	.8553	.8360	.8323
	ST(10)	.8659	.8703	.8476	.8574	.8640	.8603	.8531	.8483
	ST(8)	.8937	.9026	.8645	.8749	.8748	.8644	.8672	.8681
	ST(7)	.9354	.9453	.8896	.8947	.8919	.8888	.8940	.8656
	ST(6)	.9596	.9624	.9249	.9354	.9481	.9102	.9307	.9307
	ST(5)	1.0219	1.0365	.9921	.9995	.9695	.9484	.9795	.9788
	AVG	.7865	.7894	.7912	.7955	.8036	.7985	.7943	.7855

TABLE 33

EFFICIENCIES OF ADAPTIVE ROBUST ESTIMATES OF SCALE PARAMETER
(RELATIVE TO MAXIMUM LIKELIHOOD ESTIMATE IF POPULATION IS KNOWN)
(PHASE IV)

SAMPLE SIZE,N	SAMPLES FROM	CRITERION (A)(1)	(A)(2)	(B)(1)	(B)(2)	(D)(1)	(D)(2)	(D)(3)	(E)
10	U	1.0928	1.1206	1.0762	1.0971	1.0375	1.0684	1.0394	1.0643
	N	.8652	.8729	.8676	.8826	.8735	.8824	.8729	.8694
	D	.9088	.9143	.9022	.9091	.9028	.9094	.9012	.9024
	DS	.0149	.0149	.0149	.0149	.0149	.0149	.0149	.0149
	AS	.2082	.2132	.1952	.1995	.1888	.1921	.1908	.1929
	SB(1.5)	1.0796	1.0925	1.0729	1.0842	1.0380	1.0641	1.0396	1.0572
	SB(2.0)	.9981	1.0138	.9928	1.0011	.9599	.9768	.9632	.9839
	SB(2.5)	.9645	.9756	.9588	.9770	.9328	.9505	.9351	.9487
	SB(3.0)	.9192	.9338	.9093	.9216	.9010	.9122	.9042	.9101
	SB(3.5)	.9101	.9239	.9041	.9179	.8977	.9087	.8974	.9012
	SB(4.0)	.8981	.9043	.8863	.9029	.8811	.8892	.8841	.8878
	ST(16)	.8625	.8802	.8719	.8878	.9017	.9106	.8972	.8785
	ST(10)	.8962	.9085	.9001	.9201	.9434	.9511	.9398	.9161
	ST(8)	.9042	.9188	.9092	.9223	.9485	.9573	.9478	.9267
	ST(7)	.9713	.9902	.9792	.9938	1.0172	1.0228	1.0168	1.0027
	ST(6)	.9370	.9541	.9539	.9634	.9903	.9959	.9840	.9635
	ST(5)	1.0238	1.0346	1.0282	1.0425	1.0589	1.0680	1.0593	1.0363
	AVG	.7723	.7820	.7706	.7805	.7774	.7848	.7774	.7736
14	U	.9892	.9957	.9693	.9758	.9402	.9520	.9423	.9600
	N	.8932	.8956	.9028	.9066	.9139	.9173	.9125	.9045
	D	.9387	.9381	.9341	.9354	.9258	.9283	.9262	.9246
	DS	.0011	.0011	.0011	.0011	.0011	.0011	.0011	.0011
	AS	.0978	.0988	.0928	.0933	.0914	.0916	.0917	.0922
	SB(1.5)	1.1404	1.1395	1.1391	1.1429	1.0969	1.1104	1.1024	1.1252
	SB(2.0)	1.0651	1.0661	1.0640	1.0644	1.0272	1.0338	1.0317	1.0506
	SB(2.5)	1.0271	1.0233	1.0177	1.0181	.9904	.9926	.9972	1.0097
	SB(3.0)	.9952	.9949	.9877	.9895	.9606	.9639	.9638	.9714
	SB(3.5)	.9740	.9773	.9581	.9623	.9422	.9439	.9435	.9513
	SB(4.0)	.9504	.9498	.9413	.9443	.9231	.9259	.9260	.9278
	ST(16)	.9010	.9120	.9047	.9150	.9493	.9519	.9447	.9249
	ST(10)	.9135	.9205	.9195	.9279	.9828	.9843	.9699	.9409
	ST(8)	.9447	.9499	.9551	.9574	1.0031	1.0064	.9930	.9730
	ST(7)	.9866	.9936	.9934	1.0008	1.0485	1.0510	1.0383	1.0164
	ST(6)	1.0236	1.0344	1.0419	1.0464	1.0903	1.0928	1.0761	1.0535
	ST(5)	1.1560	1.1602	1.1620	1.1647	1.2037	1.2060	1.1913	1.1721
	AVG	.7526	.7553	.7562	.7590	.7608	.7628	.7592	.7544

TABLE 33

EFFICIENCIES OF ADAPTIVE ROBUST ESTIMATES OF SCALE PARAMETER
(RELATIVE TO MAXIMUM LIKELIHOOD ESTIMATE IF POPULATION IS KNOWN)
(PHASE IV)

SAMPLE SIZE, N	SAMPLES FROM	CRITERION (A)(1)	(A)(2)	(B)(1)	(B)(2)	(D)(1)	(D)(2)	(D)(3)	(E)
18	U	.9046	.9055	.8800	.8811	.8629	.8649	.8672	.8806
	N	.9170	.9187	.9272	.9293	.9524	.9527	.9445	.9362
	D	.9578	.9573	.9536	.9535	.9443	.9446	.9480	.9481
	DS	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AS	.0445	.0448	.0428	.0428	.0426	.0426	.0427	.0427
	SB(1.5)	1.1601	1.1513	1.1382	1.1377	1.0949	1.0963	1.1074	1.1277
	SB(2.0)	1.1029	1.0973	1.0984	1.0919	1.0552	1.0560	1.0609	1.0784
	SB(2.5)	1.0416	1.0424	1.0430	1.0443	1.0041	1.0046	1.0161	1.0239
	SB(3.0)	1.0200	1.0185	1.0254	1.0264	.9839	.9837	.9967	1.0019
	SB(3.5)	1.0067	1.0068	1.0087	1.0096	.9794	.9791	.9896	.9947
	SB(4.0)	.9970	.9977	.9907	.9928	.9730	.9727	.9795	.9825
	ST(16)	.8980	.9009	.9060	.9105	.9725	.9730	.9492	.9355
	ST(10)	.9100	.9133	.9200	.9233	.9922	.9934	.9691	.9530
	ST(8)	.9322	.9390	.9495	.9521	1.0143	1.0142	.9893	.9696
	ST(7)	.9894	.9919	.9925	.9951	1.0618	1.0615	1.0378	1.0201
	ST(6)	1.0192	1.0234	1.0376	1.0377	1.1050	1.1050	1.0806	1.0680
	ST(5)	1.1671	1.1723	1.1738	1.1770	1.2382	1.2383	1.2126	1.1943
	AVG	.7096	.7107	.7105	.7114	.7210	.7212	.7169	.7135
22	U	.8561	.8527	.8248	.8223	.8171	.8145	.8201	.8267
	N	.9320	.9334	.9374	.9380	.9748	.9745	.9699	.9648
	D	.9527	.9515	.9487	.9486	.9394	.9390	.9413	.9430
	DS	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AS	.0224	.0225	.0218	.0219	.0218	.0218	.0218	.0218
	SB(1.5)	1.1819	1.1779	1.1599	1.1551	1.1246	1.1226	1.1339	1.1451
	SB(2.0)	1.1203	1.1169	1.1201	1.1209	1.0793	1.0802	1.0940	1.1027
	SB(2.5)	1.0186	1.0196	1.0421	1.0416	1.0067	1.0066	1.0219	1.0269
	SB(3.0)	.9964	1.0003	1.0172	1.0181	.9862	.9876	1.0016	1.0038
	SB(3.5)	.9680	.9733	.9826	.9843	.9657	.9664	.9792	.9764
	SB(4.0)	.9712	.9783	.9858	.9898	.9779	.9784	.9891	.9879
	ST(16)	.8831	.8859	.9027	.9060	.9760	.9752	.9383	.9281
	ST(10)	.9072	.9089	.9193	.9215	.9969	.9960	.9636	.9517
	ST(8)	.9260	.9276	.9448	.9460	1.0207	1.0200	.9902	.9822
	ST(7)	.9839	.9859	.9885	.9905	1.0814	1.0814	1.0418	1.0274
	ST(6)	1.0713	1.0727	1.0802	1.0820	1.1390	1.1373	1.1101	1.1011
	ST(5)	1.2457	1.2460	1.2557	1.2568	1.3220	1.3218	1.2865	1.2783
	AVG	.6727	.6736	.6754	.6763	.6893	.6893	.6824	.6801

TABLE 34

EFFICIENCIES OF ADAPTIVE ROBUST ESTIMATES OF
CANONICAL SCALE PARAMETER
(RELATIVE TO MAXIMUM LIKELIHOOD ESTIMATE IF POPULATION IS KNOWN)
(PHASE IV)

SAMPLE SIZE,N	SAMPLES FROM	CRITERION (A)(1)	(A)(2)	(B)(1)	(B)(2)	(D)(1)	(D)(2)	(D)(3)	(E)
10	U	1.0921	1.1577	1.1458	1.2023	1.2043	1.2349	1.1544	1.1029
	N	.6394	.6701	.6430	.6829	.6775	.7009	.6751	.6591
	D	.7744	.7920	.7662	.7850	.7512	.7589	.7584	.7677
	DS	.2521	.2521	.2521	.2521	.2521	.2521	.2521	.2521
	AS	.3717	.3772	.5321	.5255	.6194	.6034	.5599	.5185
	SB(1.5)	.8790	.9291	.8805	.9295	.9208	.9585	.9093	.8694
	SB(2.0)	.7593	.8156	.7545	.7979	.7831	.8139	.7805	.7625
	SB(2.5)	.7184	.7652	.7114	.7709	.7427	.7779	.7379	.7162
	SB(3.0)	.6631	.7117	.6560	.6998	.7000	.7259	.6984	.6746
	SB(3.5)	.6566	.7065	.6543	.7018	.6965	.7246	.6898	.6680
	SB(4.0)	.6591	.6962	.6458	.6960	.6854	.7068	.6863	.6665
	ST(16)	.6340	.6716	.6427	.6781	.6940	.7162	.6874	.6583
	ST(10)	.6763	.7080	.6796	.7220	.7453	.7673	.7400	.7050
	ST(8)	.6822	.7146	.6863	.7186	.7398	.7654	.7404	.7101
	ST(7)	.7508	.7878	.7583	.7909	.8107	.8271	.8129	.7896
	ST(6)	.7215	.7550	.7394	.7661	.7925	.8120	.7846	.7543
	ST(5)	.8024	.8285	.8057	.8384	.8547	.8770	.8564	.8209
	AVG	.7200	.7550	.7253	.7617	.7666	.7883	.7637	.7400
14	U	1.0544	1.0734	1.1123	1.1449	1.1635	1.1837	1.1041	1.0613
	N	.6285	.6473	.6442	.6647	.6815	.7003	.6766	.6527
	D	.8066	.8124	.8001	.8110	.7688	.7781	.7797	.7863
	DS	.0292	.0292	.0102	.0102	.0589	.0589	.0292	.0292
	AS	.2777	.2736	.4222	.4117	.4593	.4600	.4308	.4200
	SB(1.5)	.8776	.9053	.9076	.9388	.9238	.9578	.9145	.8832
	SB(2.0)	.7797	.8060	.7840	.8066	.8053	.8266	.8001	.7720
	SB(2.5)	.7285	.7503	.7213	.7438	.7503	.7631	.7491	.7211
	SB(3.0)	.6991	.7210	.6898	.7114	.7137	.7318	.7075	.6828
	SB(3.5)	.6742	.7070	.6615	.6850	.6998	.7138	.6866	.6620
	SB(4.0)	.6573	.6815	.6479	.6683	.6766	.6952	.6697	.6458
	ST(16)	.6619	.6870	.6656	.6897	.7332	.7527	.7231	.6898
	ST(10)	.6777	.6932	.6851	.7042	.7762	.7916	.7518	.7098
	ST(8)	.7021	.7177	.7125	.7234	.7778	.7961	.7605	.7310
	ST(7)	.7440	.7639	.7545	.7733	.8318	.8481	.8127	.7801
	ST(6)	.7676	.7906	.7932	.8061	.8681	.8815	.8416	.8083
	ST(5)	.8775	.8926	.8871	.8989	.9561	.9731	.9293	.8995
	AVG	.7416	.7605	.7480	.7654	.7995	.8158	.7854	.7591

TABLE 34

EFFICIENCIES OF ADAPTIVE ROBUST ESTIMATES OF
CANONICAL SCALE PARAMETER
(RELATIVE TO MAXIMUM LIKELIHOOD ESTIMATE IF POPULATION IS KNOWN)
(PHASE IV)

SAMPLE SIZE,N	SAMPLES FROM	CRITERION (A)(1)	(A)(2)	(B)(1)	(B)(2)	(D)(1)	(D)(2)	(D)(3)	(E)
18	U	1.0476	1.0395	1.0954	1.1046	1.1031	1.1061	1.0831	1.0399
	N	.6478	.6596	.6636	.6752	.7302	.7447	.7068	.6843
	D	.8394	.8428	.8349	.8401	.7938	.7945	.8127	.8195
	DS	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AS	.1839	.1763	.2863	.2836	.3001	.3000	.2866	.2812
	SB(1.5)	.9223	.9280	.9443	.9602	.9323	.9383	.9224	.8867
	SB(2.0)	.7915	.8009	.8009	.8059	.7951	.8061	.7852	.7625
	SB(2.5)	.7448	.7597	.7427	.7541	.7562	.7664	.7508	.7165
	SB(3.0)	.7318	.7487	.7346	.7527	.7509	.7624	.7452	.7133
	SB(3.5)	.7118	.7275	.7037	.7154	.7302	.7388	.7187	.6950
	SB(4.0)	.7081	.7238	.6955	.7101	.7230	.7369	.7140	.6910
	ST(16)	.6531	.6625	.6590	.6679	.7567	.7680	.7158	.6925
	ST(10)	.6575	.6673	.6684	.6792	.7690	.7808	.7294	.7049
	ST(8)	.6857	.6980	.7043	.7124	.7948	.8063	.7543	.7269
	ST(7)	.7426	.7510	.7462	.7547	.8403	.8498	.7994	.7735
	ST(6)	.7511	.7610	.7709	.7761	.8661	.8746	.8245	.8054
	ST(5)	.8693	.8812	.8794	.8868	.9697	.9788	.9282	.9009
	AVG	.7432	.7532	.7534	.7623	.8157	.8249	.7897	.7677
22	U	.9908	.9910	1.0742	1.0687	1.0682	1.0744	1.0491	1.0214
	N	.6697	.6752	.6759	.6825	.7502	.7604	.7251	.7090
	D	.8412	.8446	.8346	.8367	.8032	.8021	.8178	.8258
	DS	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AS	.1293	.1234	.1869	.1855	.1907	.1913	.1895	.1875
	SB(1.5)	.9614	.9609	.9723	.9746	.9654	.9678	.9522	.9182
	SB(2.0)	.8622	.8729	.8751	.8751	.8502	.8556	.8495	.8138
	SB(2.5)	.7794	.7863	.7675	.7721	.7576	.7734	.7554	.7294
	SB(3.0)	.7719	.7851	.7672	.7701	.7587	.7674	.7565	.7304
	SB(3.5)	.7643	.7687	.7402	.7468	.7615	.7758	.7506	.7190
	SB(4.0)	.7429	.7552	.7167	.7223	.7437	.7548	.7321	.7083
	ST(16)	.6436	.6494	.6590	.6653	.7706	.7814	.7025	.6847
	ST(10)	.6707	.6736	.6788	.6848	.7890	.7977	.7349	.7165
	ST(8)	.6736	.6785	.6948	.7005	.7999	.8087	.7490	.7355
	ST(7)	.7229	.7278	.7273	.7322	.8462	.8506	.7861	.7662
	ST(6)	.7911	.7950	.7992	.8028	.8814	.8915	.8345	.8204
	ST(5)	.9080	.9102	.9181	.9216	1.0102	1.0144	.9552	.9424
	AVG	.7595	.7642	.7666	.7711	.8334	.8405	.7990	.7830

TABLE 35

EFFICIENCIES OF DEBIASED ADAPTIVE ROBUST ESTIMATES OF
SCALE PARAMETER
(RELATIVE TO DEBIASED MAXIMUM LIKELIHOOD ESTIMATE IF POPULATION IS KNOWN)
(PHASE IV)

SAMPLE SIZE,N	SAMPLES FROM	CRITERION (A)(1)	(A)(2)	(B)(1)	(B)(2)	(D)(1)	(D)(2)	(D)(3)	(E)
10	U	.8747	.8689	.8914	.8794	.8669	.8686	.8645	.8773
	N	.9413	.9617	.9511	.9718	.9987	.9926	.9989	.9765
	D	1.0155	1.0176	1.0132	1.0164	1.0254	1.0166	1.0273	1.0189
	DS	.0274	.0274	.0274	.0274	.0274	.0274	.0274	.0274
	AS	.5467	.5411	.5326	.5272	.5179	.5201	.5179	.5294
	SB(1.5)	1.1203	1.1271	1.1376	1.1373	1.1293	1.1313	1.1248	1.1225
	SB(2.0)	.9676	.9803	.9807	.9902	.9956	.9940	.9942	.9933
	SB(2.5)	.9304	.9495	.9441	.9600	.9658	.9653	.9634	.9549
	SB(3.0)	.9177	.9378	.9239	.9428	.9661	.9660	.9654	.9504
	SB(3.5)	.8994	.9243	.9098	.9322	.9471	.9417	.9468	.9323
	SB(4.0)	.9154	.9369	.9276	.9499	.9654	.9642	.9657	.9503
	ST(16)	.8943	.9159	.9090	.9298	.9594	.9515	.9630	.9301
	ST(10)	.8720	.8930	.8886	.9057	.9387	.9289	.9400	.9088
	ST(8)	.8786	.8961	.8890	.9030	.9405	.9284	.9414	.9137
	ST(7)	.9185	.9383	.9330	.9501	.9831	.9765	.9838	.9619
	ST(6)	.8988	.9154	.9178	.9312	.9602	.9498	.9626	.9360
	ST(5)	.9443	.9579	.9542	.9676	.9915	.9804	.9922	.9663
	AVG	.8395	.8528	.8489	.8603	.8783	.8727	.8789	.8629
14	U	.8123	.7981	.8116	.7998	.7705	.7690	.7747	.7906
	N	.8995	.9128	.9286	.9377	.9841	.9821	.9758	.9521
	D	1.0129	1.0129	1.0122	1.0110	1.0152	1.0070	1.0148	1.0058
	DS	.0017	.0017	.0018	.0018	.0017	.0017	.0017	.0017
	AS	.2399	.2383	.2303	.2293	.2284	.2280	.2287	.2314
	SB(1.5)	1.1224	1.1279	1.1504	1.1503	1.1319	1.1345	1.1313	1.1416
	SB(2.0)	.9215	.9423	.9564	.9696	.9617	.9699	.9600	.9619
	SB(2.5)	.8948	.9122	.9157	.9313	.9403	.9484	.9381	.9338
	SB(3.0)	.8643	.8834	.8863	.9046	.9110	.9155	.9109	.9025
	SB(3.5)	.8766	.8968	.8908	.9072	.9245	.9285	.9295	.9200
	SB(4.0)	.8667	.8858	.8915	.9079	.9239	.9269	.9205	.9084
	ST(16)	.8530	.8706	.8677	.8828	.9320	.9263	.9267	.8990
	ST(10)	.8261	.8401	.8367	.8499	.9037	.9002	.8975	.8664
	ST(8)	.8629	.8738	.8787	.8868	.9381	.9353	.9334	.9088
	ST(7)	.8738	.8819	.8840	.8918	.9400	.9326	.9366	.9123
	ST(6)	.9054	.9164	.9214	.9294	.9693	.9662	.9629	.9389
	ST(5)	.9958	1.0014	1.0023	1.0077	1.0446	1.0377	1.0391	1.0180
	AVG	.7746	.7826	.7896	.7962	.8147	.8127	.8112	.7981

AD-A062 436 AIR FORCE FLIGHT DYNAMICS LAB WRIGHT-PATTERSON AFB OHIO F/G 12/1
ADAPTIVE ROBUST ESTIMATION OF LOCATION AND SCALE PARAMETERS OF --ETC(U)
SEP 78 H L HARTER, A H MOORE, T F CURRY

UNCLASSIFIED AFFDL-TR-78-128 NL

2 OF 2
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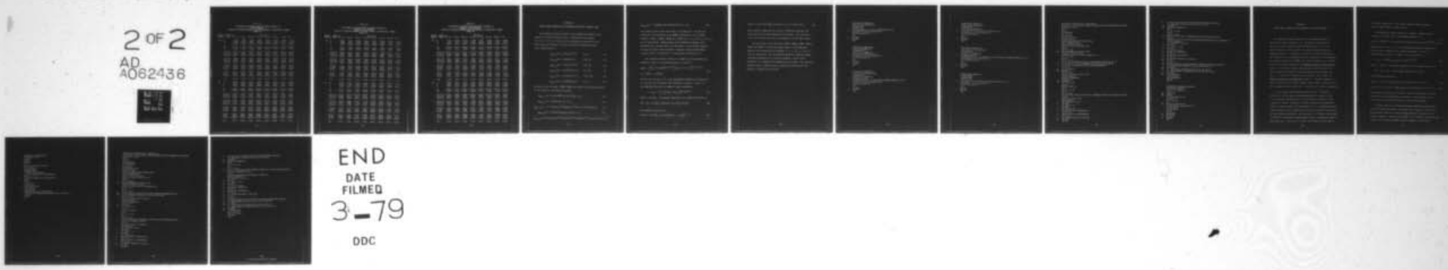

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TABLE 35

EFFICIENCIES OF DEBIASED ADAPTIVE ROBUST ESTIMATES OF
SCALE PARAMETER
(RELATIVE TO DEBIASED MAXIMUM LIKELIHOOD ESTIMATE IF POPULATION IS KNOWN)
(PHASE IV)

SAMPLE SIZE,N	SAMPLES FROM	CRITERION (A)(1)	(A)(2)	(B)(1)	(B)(2)	(D)(1)	(D)(2)	(D)(3)	(E)
18	U	.7153	.7082	.7018	.6946	.6833	.6803	.6878	.7022
	N	.8708	.8834	.8997	.9118	.9592	.9597	.9437	.9257
	D	1.0103	1.0096	1.0061	1.0059	1.0123	1.0084	1.0113	1.0068
	DS	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AS	.0919	.0919	.0895	.0892	.0899	.0896	.0900	.0902
	SB(1.5)	1.0404	1.0436	1.0577	1.0653	1.0418	1.0547	1.0456	1.0562
	SB(2.0)	.8234	.8387	.8568	.8674	.8644	.8775	.8622	.8650
	SB(2.5)	.8075	.8276	.8474	.8633	.8603	.8698	.8610	.8577
	SB(3.0)	.7902	.8080	.8377	.8506	.8410	.8499	.8438	.8384
	SB(3.5)	.8255	.8461	.8620	.8797	.8847	.8973	.8868	.8813
	SB(4.0)	.8229	.8414	.8565	.8734	.8938	.9018	.8896	.8818
	ST(16)	.8118	.8209	.8329	.8433	.9134	.9127	.8686	.8711
	ST(10)	.8142	.8212	.8328	.8382	.9085	.9089	.8871	.8691
	ST(8)	.8115	.8210	.8316	.8361	.9008	.8977	.8755	.8550
	ST(7)	.8414	.8469	.8516	.8552	.9191	.9160	.8989	.8812
	ST(6)	.8647	.8722	.8855	.8895	.9482	.9444	.9290	.9155
	ST(5)	.9631	.9687	.9733	.9771	1.0305	1.0275	1.0094	.9923
	AVG	.7016	.7076	.7139	.7183	.7416	.7415	.7323	.7250
22	U	.6621	.6520	.6325	.6273	.6278	.6234	.6322	.6390
	N	.8640	.8756	.8910	.8944	.9656	.9689	.9516	.9419
	D	.9915	.9896	.9883	.9883	.9888	.9860	.9854	.9846
	DS	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AS	.0384	.0385	.0378	.0378	.0379	.0378	.0379	.0379
	SB(1.5)	.8490	.8596	.8726	.8811	.8621	.8729	.8680	.8713
	SB(2.0)	.7241	.7356	.7537	.7716	.7637	.7809	.7681	.7683
	SB(2.5)	.6968	.7126	.7575	.7725	.7699	.7808	.7736	.7722
	SB(3.0)	.7325	.7508	.7877	.8063	.8108	.8243	.8148	.8121
	SB(3.5)	.7272	.7487	.7826	.7949	.8135	.8239	.8162	.8093
	SB(4.0)	.7633	.7822	.8183	.8360	.8560	.8659	.8579	.8521
	ST(16)	.7879	.7959	.8182	.8253	.8995	.8984	.8637	.8516
	ST(10)	.7921	.7970	.8123	.8164	.8902	.8868	.8587	.8469
	ST(8)	.7967	.8005	.8164	.8186	.8858	.8839	.8591	.8507
	ST(7)	.8275	.8312	.8362	.8385	.9177	.9186	.8847	.8720
	ST(6)	.8940	.8973	.9063	.9097	.9618	.9555	.9369	.9286
	ST(5)	1.0164	1.0193	1.0315	1.0337	1.0916	1.0908	1.0611	1.0534
	AVG	.6451	.6493	.6581	.6618	.6846	.6858	.6750	.6710

TABLE 36

EFFICIENCIES OF DEBIASED ADAPTIVE ROBUST ESTIMATES OF
CANONICAL SCALE PARAMETER
(RELATIVE TO DEBIASED MAXIMUM LIKELIHOOD ESTIMATE IF POPULATION IS KNOWN)
(PHASE IV)

SAMPLE SIZE,N	SAMPLES FROM	CRITERION (A)(1)	(A)(2)	(B)(1)	(B)(2)	(D)(1)	(D)(2)	(D)(3)	(E)
10	U	.5811	.5671	.6638	.6381	.7641	.7016	.7002	.6216
	N	.7828	.8076	.7901	.8256	.8821	.8922	.8703	.8137
	D	.9152	.9276	.9074	.9219	.9207	.9321	.9172	.9114
	DS	.0815	.0815	.0815	.0815	.0815	.0815	.0815	.0815
	AS	.1861	.1787	.2959	.2709	.3804	.3460	.3320	.3017
	SB(1.5)	.9083	.9123	.9274	.9223	1.0253	.9792	1.0061	.8965
	SB(2.0)	.8250	.8517	.8161	.8240	.8952	.8806	.8823	.8112
	SB(2.5)	.8041	.8303	.8012	.8325	.8903	.8831	.8748	.7954
	SB(3.0)	.7675	.8045	.7620	.7868	.8639	.8514	.8572	.7805
	SB(3.5)	.7601	.7983	.7587	.7902	.8536	.8540	.8400	.7728
	SB(4.0)	.7853	.8089	.7735	.8140	.8744	.8689	.8610	.7938
	ST(16)	.7351	.7667	.7489	.7786	.8516	.8647	.8334	.7670
	ST(10)	.7298	.7537	.7388	.7710	.8428	.8558	.8296	.7664
	ST(8)	.7278	.7521	.7339	.7558	.8233	.8388	.8165	.7630
	ST(7)	.7700	.7978	.7827	.8060	.8678	.8760	.8605	.8158
	ST(6)	.7340	.7612	.7574	.7756	.8404	.8517	.8259	.7750
	ST(5)	.7776	.7959	.7829	.8059	.8544	.8695	.8488	.7988
	AVG	.7610	.7830	.7737	.7953	.8552	.8601	.8431	.7897
14	U	.6034	.5747	.7009	.6830	.7847	.7427	.7069	.6414
	N	.7282	.7409	.7423	.7570	.8339	.8435	.8155	.7597
	D	.9130	.9167	.9058	.9132	.8963	.9060	.8980	.8929
	DS	.0137	.0137	.0064	.0064	.0201	.0201	.0137	.0137
	AS	.1721	.1632	.3351	.3116	.4156	.4077	.3723	.3540
	SB(1.5)	.9741	.9641	1.0178	1.0092	1.0690	1.0486	1.0432	.9590
	SB(2.0)	.8698	.8722	.8731	.8651	.9278	.9067	.9197	.8445
	SB(2.5)	.8196	.8258	.8083	.8070	.8753	.8603	.8703	.7992
	SB(3.0)	.7921	.7980	.7755	.7765	.8328	.8286	.8124	.7514
	SB(3.5)	.7735	.7885	.7458	.7555	.8308	.8273	.8035	.7408
	SB(4.0)	.7629	.7729	.7427	.7514	.8105	.8116	.7989	.7394
	ST(16)	.7045	.7255	.7101	.7291	.8219	.8334	.7995	.7415
	ST(10)	.6748	.6864	.6820	.6950	.8003	.8076	.7673	.7095
	ST(8)	.6968	.7068	.7113	.7169	.8062	.8155	.7776	.7342
	ST(7)	.7076	.7205	.7158	.7289	.8160	.8268	.7891	.7451
	ST(6)	.7182	.7356	.7410	.7494	.8320	.8390	.7999	.7577
	ST(5)	.7827	.7923	.7918	.7983	.8723	.8820	.8442	.8079
	AVG	.7475	.7576	.7548	.7629	.8401	.8454	.8156	.7698

TABLE 36

EFFICIENCIES OF DEBIASED ADAPTIVE ROBUST ESTIMATES OF CANONICAL SCALE PARAMETER (RELATIVE TO DEBIASED MAXIMUM LIKELIHOOD ESTIMATE IF POPULATION IS KNOWN) (PHASE IV)

SAMPLE SIZE,N	SAMPLES FROM	CRITERION (A)(1)	(A)(2)	(B)(1)	(B)(2)	(D)(1)	(D)(2)	(D)(3)	(E)
18	U	.6441	.6005	.7523	.7228	.7934	.7480	.7615	.6866
	N	.7103	.7174	.7255	.7306	.8403	.8484	.7966	.7537
	D	.9192	.9216	.9143	.9174	.8929	.8948	.9022	.9023
	DS	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AS	.1367	.1266	.3388	.3250	.4118	.4031	.3664	.3497
	SB(1.5)	1.0749	1.0447	1.1267	1.1073	1.1300	1.0920	1.1054	1.0178
	SB(2.0)	.8616	.8410	.8630	.8428	.8692	.8527	.8536	.8009
	SB(2.5)	.7927	.7926	.7890	.7847	.8322	.8253	.8207	.7525
	SB(3.0)	.7861	.7886	.7822	.7846	.8226	.8187	.8057	.7458
	SB(3.5)	.7925	.7941	.7715	.7709	.8347	.8276	.8111	.7607
	SB(4.0)	.7823	.7870	.7544	.7581	.8170	.8172	.7961	.7490
	ST(16)	.6579	.6635	.6632	.6693	.7939	.8013	.7383	.7044
	ST(10)	.6372	.6437	.6483	.6553	.7725	.7794	.7222	.6894
	ST(8)	.6370	.6454	.6528	.6577	.7607	.7672	.7135	.6802
	ST(7)	.6684	.6729	.6722	.6772	.7764	.7817	.7313	.7012
	ST(6)	.6647	.6709	.6831	.6847	.7838	.7898	.7386	.7159
	ST(5)	.7367	.7444	.7463	.7510	.8378	.8428	.7960	.7680
	AVG	.7166	.7205	.7274	.7304	.8163	.8189	.7790	.7463
22	U	.6087	.5847	.7859	.7501	.8239	.8048	.7878	.7344
	N	.7095	.7113	.7155	.7194	.8302	.8356	.7854	.7588
	D	.9027	.9047	.8957	.8972	.8767	.8770	.8826	.8869
	DS	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	AS	.1255	.1142	.3841	.3643	.4407	.4402	.4268	.4089
	SB(1.5)	1.0220	.9885	1.0585	1.0326	1.0405	1.0130	1.0203	.9568
	SB(2.0)	.8728	.8619	.8871	.8655	.8534	.8379	.8477	.7907
	SB(2.5)	.8047	.7962	.7919	.7831	.7843	.7844	.7754	.7342
	SB(3.0)	.8121	.8122	.8037	.7952	.8046	.8025	.7942	.7532
	SB(3.5)	.7934	.7874	.7626	.7605	.8045	.8067	.7829	.7369
	SB(4.0)	.7813	.7851	.7504	.7483	.8031	.8049	.7806	.7432
	ST(16)	.6325	.6354	.6476	.6517	.7838	.7901	.7018	.6783
	ST(10)	.6287	.6301	.6385	.6424	.7614	.7673	.6998	.6785
	ST(8)	.6096	.6125	.6287	.6318	.7384	.7437	.6841	.6684
	ST(7)	.6353	.6380	.6398	.6429	.7586	.7607	.6988	.6787
	ST(6)	.6830	.6852	.6906	.6926	.7735	.7803	.7273	.7126
	ST(5)	.7560	.7566	.7656	.7677	.8547	.8572	.8022	.7894
	AVG	.7116	.7121	.7204	.7215	.8037	.8065	.7605	.7397

APPENDIX B

RANDOM NUMBER GENERATION AND PARAMETER ESTIMATION--SYMMETRIC BETA

The probability density function of the standardized symmetric beta population is given by Equation (22). Substitution of the values (1.5, 2.0, 2.5, 3.0, 3.5 and 4.0) of the parameter p considered in the Monte Carlo study (Phases III and IV) yields the following equations (after simplification):

$$f_{SB(1.5)}(x) = (1/2\pi)(4-x^2)^{1/2}, \qquad (-2,\ 2) \tag{34}$$

$$f_{SB(2.0)}(x) = (3/20\sqrt{5})(5-x^2), \qquad (-\sqrt{5},\ \sqrt{5}) \tag{35}$$

$$f_{SB(2.5)}(x) = (2/27\pi)(6-x^2)^{3/2}, \qquad (-\sqrt{6},\ \sqrt{6}) \tag{36}$$

$$f_{SB(3.0)}(x) = (15/784\sqrt{7})(7-x^2)^2, \qquad (-\sqrt{7},\ \sqrt{7}) \tag{37}$$

$$f_{SB(3.5)}(x) = (1/160\pi)(8-x^2)^{5/2}, \qquad (-2\sqrt{2},\ 2\sqrt{2}) \tag{38}$$

$$f_{SB(4.0)}(x) = (35/69984)(9-x^2)^3, \qquad (-3,\ 3) \tag{39}$$

Integration over the range $(-\sqrt{2p+1},\ \sqrt{2p+1})$ then yields the following equations for the cumulative distribution functions:

$$F_{SB(1.5)}(x) = (1/2\pi)[x\sqrt{4-x^2}/2+2\sin^{-1}(x/2)] + 1/2 \tag{40}$$

$$F_{SB(2.0)}(x) = (3/20\sqrt{5})(5x-x^3/3) + 1/2 \tag{41}$$

$$F_{SB(2.5)}(x) = (2/27\pi)[x(6-x^2)^{3/2}/4+9x(6-x^2)^{1/2}/4+27\sin^{-1}(x/\sqrt{6})/2]+1/2 \tag{42}$$

$$F_{SB(3.0)}(x) = (15/784\sqrt{7})(49x-14x^3/3+x^5/5) + 1/2 \tag{43}$$

$$F_{SB(3.5)}(x)=(1/160\pi)[x(8-x^2)^{5/2}/6+5x(8-x^2)^{3/2}/3+20x(8-x^2)^{1/2}+160\sin^{-1}(x/2\sqrt{2})]+1/2 \tag{44}$$

$$F_{SB(4.0)}(x) = (35/69984)(729x-81x^3+27x^5/5-x^7/7) + 1/2 \tag{45}$$

The canonical scale factors were found, by setting $F(x) = .975$ and then solving for x (by iteration on the HP9830A calculator), to be 1.75668, 1.81435, 1.84812, 1.86984, 1.88482 and 1.89569 for p = 1.5, 2.0, 2.5, 3.0, 3.5 and 4.0 respectively. Random numbers from standardized symmetric beta populations for the same values of p were found, on the CDC 6600 computer, by using the library subroutine RANF to generate uniform random numbers r between 0 and 1, setting $F(x) = r$, and solving iteratively for x.

The probability density function of a symmetric beta population with parameter p, mean μ and standard deviation σ is given by

$$f_{SB}(x) = \left[\Gamma(2p)/\Gamma^2(p)(2\sqrt{2p+1})^{2p-1}\right]\left[(2p+1)\sigma^2-(x-\mu)^2\right]^{p-1}/\sigma^{2p-1}, \tag{46}$$

$$(\mu-\sigma\sqrt{2p+1},\ \mu+\sigma\sqrt{2p+1})$$

Equation (22) for the p.d.f. of the standardized symmetric beta population is a special case of Equation (46) obtained by setting $\mu = 0$ and $\sigma = 1$. The likelihood function of a sample of size n is given by

$$L = L_{SB} = C\ \Pi_{i=1}^{n}\left[(2p+1)\sigma^2-(x_i-\mu)^2\right]^{p-1}/\sigma^{n(2p-1)} \tag{47}$$

where C = constant. The natural logarithm of the likelihood function is

$$\ell nL = \ell nC + (p-1)\sum_{i=1}^{n} \ell n\left[(2p+1)\sigma^2-(x_i-\mu)^2\right]-n(2p-1)\ell n\sigma \tag{48}$$

The likelihood equations are

$$\partial\ell nL/\partial\mu = 2(p-1)\sum_{i=1}^{n} \{(x_i-\mu)/\left[(2p+1)\sigma^2 - (x_i-\mu)\right]^2\} = 0 \tag{49}$$

$$\partial \ell nL/\partial\sigma = 2(p-1)(2p+1)\sigma\sum_{i=1}^{n}\{1/[(2p+1)\sigma^2-(x_i-\mu)^2]\}-n(2p-1)/\sigma=0 \qquad (50)$$

These equations apparently do not have a closed-form solution, and hence they must be solved numerically by iteration. This iteration, by the rule of false position, was performed on the CDC 6600 computer.

Listings follow of the subroutines SBRN15, SBRN20, SBRN25, SBRN30, SBRN35 and SBRN40 for generating random numbers from standardized symmetric beta populations with parameter p = 1.5, 2.0, 2.5, 3.0, 3.5 and 4.0, respectively, and the subroutine IESBP for iterative maximum likelihood estimation of the location parameter μ and the scale parameter σ of a symmetric beta population with p known. The subroutine SORTSUB called in IESBP and used in ordering the observations from smallest to largest is also listed.

```
      SUBROUTINE SBRN15(X)
      C=1./(2.*3.1415926536)
      Y=RANF(DUM)
      X=4.*(Y-.5)
1     F=C*(X*SQRT(4.-X**2)/2.+2*ASIN(X/2.))+.5
      IF(ABS(F-Y).LT.1.E-8)GO TO 2
      Z=(Y-.5)*X/(F-.5)
      X=Z
      GO TO 1
2     RETURN
      END

      SUBROUTINE SBRN20(X)
      C=3./(20.*SQRT(5.))
      Y=RANF(DUM)
      X=2.*(Y-.5)*SQRT(5.)
1     F=C*(5.*X-(X**3)/3.)+.5
      IF(ABS(F-Y).LT.1.E-8)GO TO 2
      Z=(Y-.5)*X/(F-.5)
      X=Z
      GO TO 1
2     RETURN
      END

      SUBROUTINE SBRN25(X)
      C=2./(27.*3.1415926536)
      Y=RANF(DUM)
      X=2.*(Y-.5)*SQRT(6.)
1     R=SQRT(6.-X**2)
      F=C*(X*(R**3)/4.+2.25*X*R+13.5*ASIN(X/SQRT(6.)))+.5
      IF(ABS(F-Y).LT.1.E-8)GO TO 2
      Z=(Y-.5)*X/(F-.5)
      X=Z
      GO TO 1
2     RETURN
      END
```

```
      SUBROUTINE SBRN30(X)
      C=15./(784.*SQRT(7.))
      Y=RANF(DUM)
      X=2.*(Y-.5)*SQRT(7.)
1     F=C*(49.*X-14.*(X**3)/3.+(X**5)/5.)+.5
      IF(ABS(F-Y).LT.1.E-8)GO TO 2
      Z=(Y-.5)*X/(F-.5)
      X=Z
      GO TO 1
2     RETURN
      END


      SUBROUTINE SBRN35(X)
      C=1./(160.*3.1415926536)
      Y=RANF(DUM)
      X=2.*(Y-.5)*SQRT(8.)
1     R=SQRT(8.-X**2)
      F=C*(X*(R**5)/6.+5.*X*(R**3)/3.+20.*X*R+160.*ASIN(X/SQRT(8.)))+.5
      IF(ABS(F-Y).LT.1.E-8)GO TO 2
      Z=(Y-.5)*X/(F-.5)
      X=Z
      GO TO 1
2     RETURN
      END


      SUBROUTINE SBRN40(X)
      C=35./69984.
      Y=RANF(DUM)
      X=6.*(Y-.5)
1     F=C*(729.*X-81.*(X**3)+5.4*(X**5)-(X**7)/7.)+.5
      IF(ABS(F-Y).LT.1.E-8)GO TO 2
      Z=(Y-.5)*X/(F-.5)
      X=Z
      GO TO 1
2     RETURN
      END
```

```
      SUBROUTINE IESBP(P,N,T ,EMUL,SIGL)
      DIMENSION EM(50),SIG(50),DLS(50),DLM(50),T(24),SIGMA(50),EMU(50)
      DIMENSION EL(50)
      ST=0.
      ST2=0.
      EN=FLOAT(N)
      DO 1 I=1,N
      ST=ST+T(I)
1     ST2=ST2+T(I)**2
      EMU(1)=ST/EN
      SIGMA(1)=SQRT(EN*ST2-ST**2)/EN
      CALL SORTSUB(T,N)
      EMU(2)=(T(1)+T(N))/2.
      EMU(3)=(T(N/2)+T(N/2+1))/2.
      SUM=0.
      DO 26 I=1,N
26    SUM=SUM+ABS(T(I)-EMU(3))/EN
      SIGMA(3)=SQRT(2.)*SUM
      SIGMA(2)=(T(N)-T(1))/(2.*SQRT(3.))
      DO 25 J=1,3
      S3=0.
      DO 24 I=1,N
      IF((2.*P+1.)*SIGMA(J)**2.LT.(T(I)-EMU(J))**2)GO TO 27
24    S3=S3+ALOG((2.*P+1.)*SIGMA(J)**2-(T(I)-EMU(J))**2)
      EL(J)=(P-1.)*S3-EN*(2.*P-1.)*ALOG(SIGMA(J))
      GO TO 25
27    EL(J)=-9.E99
25    CONTINUE
      DO 28 J=2,3
      IF(EL(J).LE.EL(1))GO TO 28
      EMU(1)=EMU(J)
      SIGMA(1)=SIGMA(J)
      EL(1)=EL(J)
28    CONTINUE
      DO 22 J=2,50
      JJ=J-1
      EMU(J)=EMU(JJ)
      KS=0
      DO 10 K=1,50
      S1=0.
      DO 2 I=1,N
2     S1=S1+(T(I)-EMU(J))/((2.*P+1.)*(SIGMA(JJ)**2)-(T(I)-EMU(J))**2)
      KK=K-1
      DLM(K)=2.*(P-1.)*S1
      EM(K)=EMU(J)
      IF (DLM(K)) 3,11,4
3     KS=KS-1
      IF (KS+K) 7,5
4     KS=KS+1
      IF (KS-K) 7,6
5     EMU(J)=EM(K)-.01*SIGMA(JJ)
      GO TO 10
6     EMU(J)=EM(K)+.01*SIGMA(JJ)
      GO TO 10
7     IF (DLM(K)*DLM(KK)) 9,11,8
8     KK=KK-1
      GO TO 7
```

```
9     EMU(J)=EM(K)+DLM(K)*(EM(K)-EM(KK))/(DLM(KK)-DLM(K))
      IF (ABS(EMU(J)-EM(K)).LE.1.E-6) GO TO 11
10    CONTINUE
11    SIGMA(J)=SIGMA(JJ)
      KS=0
      DO 20 K=1,50
      S2=0.
      DO 12 I=1,N
12    S2=S2+1./((2.*P+1.)*(SIGMA(J)**2)-(T(I)-EMU(J))**2)
      KK=K-1
      DLS(K)=2.*(P-1.)*(2.*P+1.)*SIGMA(J)*S2-EN*(2.*P-1.)/SIGMA(J)
      SIG(K)=SIGMA(J)
      IF (DLS(K)) 13,21,14
13    KS=KS-1
      IF (KS+K) 17,15
14    KS=KS+1
      IF (KS-K) 17,16
15    SIGMA(J)=.99*SIG(K)
      GO TO 20
16    SIGMA(J)=1.01*SIG(K)
      GO TO 20
17    IF (DLS(K)*DLS(KK)) 19,21,18
18    KK=KK-1
      GO TO 17
19    SIGMA(J)=SIG(K)+DLS(K)*(SIG(K)-SIG(KK))/(DLS(KK)-DLS(K))
      IF (ABS(SIGMA(J)-SIG(K)).LE.1.E-6) GO TO 21
20    CONTINUE
21    IF (ABS(EMU(J)-EMU(JJ)).GT.1.E-6) GO TO 22
      IF (ABS(SIGMA(J)-SIGMA(JJ)).LE.1.E-6) GO TO 23
22    CONTINUE
23    JL=MIN0(J,50)
      EMUL=EMU(JL)
      SIGL=SIGMA(JL)
      RETURN
      END

      SUBROUTINE SORTSUB(X,ISIZE)
      DIMENSION X(ISIZE)
      DO 10 L=1,ISIZE,L
      M=2*L-1
10    CONTINUE
20    M=M/2
      IF(M.EQ.0)GO TO 70
      K=ISIZE-M
      DO 60 J=1,K
      L=J
30    IF(L.LT.1)GO TO 60
      IF(X(L+M).GE.X(L))GO TO 60
      X(L)=X(L+M)
      X(L+M)=TEMP
      L=L-M
      GO TO 30
60    CONTINUE
      GO TO 20
70    RETURN
      END
```

APPENDIX C

RANDOM NUMBER GENERATION AND PARAMETER ESTIMATION--STUDENT t

The probability density function of the standardized Student t population is given by Equation (22). The Student t population in its usual form has standard deviation $\sqrt{\nu/(\nu-2)}$, where $\nu(>2)$ is the number of degrees of freedom, while the standardized population has standard deviation 1. Therefore, to obtain the canonical scale factors (97.5% points) of the standardized Student t population, one must multiply the usual 97.5% points by $\sqrt{(\nu-2)/\nu}$. The usual 97.5% points (to 3 decimal places) can be read from Table 12 of Biometrika Tables for Statisticians, Volume I. For greater accuracy, one can take the square roots of the upper 5% points (two-sided) of the Fisher-Snedecor F distribution for $\nu_1=1$, $\nu_2 = \nu$ degrees of freedom, which are given to 4DP in Table 5 of Biometrika Tables for Statisticians, Volume II. The latter method was used to obtain values of 1.9830, 1.9929, 1.9971, 1.9985, 1.9979 and 1.9912 for the canonical scale factors for ν = 16, 10, 8, 7, 6 and 5, respectively.

If x is normally distributed with mean m, then $t = \sqrt{n}(\bar{x}-m)/s$, where $\bar{x}$ and s are the mean and standard deviation of a sample of size $n = \nu+1$ from the distribution of x, is distributed as Student's t with ν degrees of freedom. Therefore, in order to generate one standardized Student t random variable, one must generate $n = \nu + 1$ standardized normal random variables by use of Equations (17) and (18), calculate their mean $\bar{x}$ and their standard deviation s, then calculate $t = \sqrt{n}\,(\bar{x}-m)/s$, and multiply by $\sqrt{(\nu-2)/\nu}$ to standardize. Random numbers from the standardized Student t populations for ν = 16, 10, 8, 7, 6 and 5 were found on the CDC 6600 by

this method, making use of the library subroutine RANF to generate uniform random numbers r between 0 and 1.

The probability density function of a Student t population with ν degrees of freedom, mean μ and standard deviation σ is

$$f_{ST}(x) = \sqrt{1/(\nu-2)\sigma^2}\ \{\Gamma(\nu+1)/2]/\Gamma(1/2)\Gamma(\nu/2)\}[1+(x-\mu)^2/(\nu-2)\sigma^2]^{-(\nu+1)/2},\ (-\infty,\ \infty) \tag{51}$$

Equation (23) for the p.d.f. of the standardized Student t population is a special case of Equation (51) obtained by setting $\mu = 0$ and $\sigma = 1$. The likelihood function of a sample of size n is given by

$$L = L_{ST} = (C/\sigma^n)\Pi_{i=1}^n\ [1+(x_i-\mu)^2/(\nu-2)\sigma^2]^{-(\nu+1)/2} \tag{52}$$

where C = constant. The natural logarithm of the likelihood function is

$$\ell nL = \ell nC - n\ell n\sigma - [(\nu+1)/2]\textstyle\sum_{i=1}^n\ \ell n[1+(x_i-\mu)^2/(\nu-2)\sigma^2] \tag{53}$$

The likelihood equations are

$$\partial\ell nL/\partial\mu = [(\nu+1)/(\nu-2)]\textstyle\sum_{i=1}^n\{[(x_i-\mu)/\sigma^2]/[1+(x_i-\mu)^2/(\nu-2)\sigma^2]\} = 0 \tag{54}$$

$$\partial\ell nL/\partial\sigma = -n/\sigma + [(\nu+1)/(\nu-2)]\textstyle\sum_{i=1}^n\{[(x_i-\mu)^2/\sigma^3]/[1+(x_i-\mu)^2/(\nu-2)\sigma^2]\} = 0 \tag{55}$$

These equations apparently do not have a closed form solution, and hence they must be solved numerically by iteration. This iteration, by the rule of false position, was performed on the CDC 6600 computer.

Listings follow of the subroutine STRN for generating random numbers from the standardized Student t population with ν degrees of freedom and the subroutine IESTP for iterative maximum likelihood estimation of the location parameter μ and the scale parameter σ of a Student t population with ν known. The subroutine SORTSUB listed in Appendix B is also used.

```
      SUBROUTINE STRN(XNU,Z)
      DIMENSION X(20)
      NU=XNU
      N=NU+1
      NN=N+1
      J=N
      IF((NN/2-N/2).EQ.0)J=NU
      DO 1 I=1,J,2
      R1=RANF(DUM)
      R2=RANF(DUM)
      Y=SQRT(-2.*ALOG(R2))
      X(I)=Y*COS(R1*2.*3.1415926536)
      II=I+1
1     X(II)=Y*SIN(R1*2.*3.1415926536)
      SX=0.
      SX2=0.
      DO 2 I=1,N
      SX=SX+X(I)
2     SX2=SX2+X(I)**2
      EN=FLOAT(N)
      XBAR=SX/EN
      SIGMA=SQRT(EN*SX2-SX**2)/EN
      Z=SQRT(EN)*(XBAR/SIGMA)*SQRT((EN-3.)/(EN-1.))
      RETURN
      END
```

```
      SUBROUTINE IESTP(NU,N,T ,EMUL,SIGL)
      DIMENSION EM(50),SIG(50),DLM(50),DLS(50),T(24),SIGMA(50),EMU(50)
      DIMENSION EL(50)
      ST=0.
      ST2=0.
      EN=FLOAT(N)
      ENU=FLOAT(NU)
      DO 1 I=1,N
      ST=ST+T(I)
1     ST2=ST2+T(I)**2
      EMU(1)=ST/EN
      SIGMA(1)=SQRT(EN*ST2-ST**2)/EN
      CALL SORTSUB(T,N)
      EMU(2)=(T(1)+T(N))/2.
      EMU(3)=(T(N/2)+T(N/2+1))/2.
      SUM=0.
      DO 26 I=1,N
26    SUM=SUM+ABS(T(I)-EMU(3))/EN
      SIGMA(3)=SQRT(2.)*SUM
      SIGMA(2)=(T(N)-T(1))/(2.*SQRT(3.))
      DO 25 J=1,3
      S3=0.
      DO 24 I=1,N
24    S3=S3+ALOG(1.+(T(I)-EMU(J))**2/((ENU-2.)*SIGMA(J)**2))
25    EL(J)=-EN*ALOG(SIGMA(J))-((ENU+1.)/2.)*S3
      DO 28 J=2,3
      IF(EL(J).LE.EL(1))GO TO 28
      EMU(1)=EMU(J)
      SIGMA(1)=SIGMA(J)
      EL(1)=EL(J)
28    CONTINUE
      DO 22 J=2,50
      JJ=J-1
      EMU(J)=EMU(JJ)
      KS=0
      DO 10 K=1,50
      S=0.
      DO 2 I=1,N
2     S=S+((T(I)-EMU(J))/SIGMA(JJ)**2)/(1.+((T(I)-EMU(J))**2)/
     1((ENU-2.)*SIGMA(JJ)**2))
      KK=K-1
      DLM(K)=S*(ENU+1.)/(ENU-2.)
      EM(K)=EMU(J)
      IF (DLM(K)) 3,11,4
3     KS=KS-1
      IF (KS+K) 7,5
4     KS=KS+1
      IF (KS-K) 7,6
5     EMU(J)=EM(K)-.01*SIGMA(JJ)
      GO TO 10
6     EMU(J)=EM(K)+.01*SIGMA(JJ)
      GO TO 10
7     IF (DLM(K)*DLM(KK)) 9,11,8
8     KK=KK-1
      GO TO 7
```

```
9     EMU(J)=EM(K)+DLM(K)*(EM(K)-EM(KK))/(DLM(KK)-DLM(K))
      IF (ABS(EMU(J)-EM(K)).LE.1.E-6) GO TO 11
10    CONTINUE
11    SIGMA(J)=SIGMA(JJ)
      KS=0
      DO 20 K=1,50
      S=0.
      DO 12 I=1,N
12    S=S+(((T(I)-EMU(J))**2)/SIGMA(J)**3)/(1.+((T(I)-EMU(J))**2)/
     1((ENU-2.)*SIGMA(J)**2))
      KK=K-1
      DLS(K)=-EN/SIGMA(J)+S*(ENU+1.)/(ENU-2.)
      SIG(K)=SIGMA(J)
      IF (DLS(K)) 13,21,14
13    KS=KS-1
      IF (KS+K) 17,15
14    KS=KS+1
      IF (KS-K) 17,16
15    SIGMA(J)=.99*SIG(K)
      GO TO 20
16    SIGMA(J)=1.01*SIG(K)
      GO TO 20
17    IF (DLS(K)*DLS(KK)) 19,21,18
18    KK=KK-1
      GO TO 17
19    SIGMA(J)=SIG(K)+DLS(K)*(SIG(K)-SIG(KK))/(DLS(KK)-DLS(K))
      IF (ABS(SIGMA(J)-SIG(K)).LE.1.E-6) GO TO 21
20    CONTINUE
21    IF (ABS(EMU(J)-EMU(JJ)).GT.1.E-6) GO TO 22
      IF (ABS(SIGMA(J)-SIGMA(JJ)).LE.1.E-6) GO TO 23
22    CONTINUE
23    JL=MIN0(J,50)
      EMUL=EMU(JL)
      SIGL=SIGMA(JL)
      RETURN
      END
```


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